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3D Multiphase Piecewise Constant Level Set Method Based on Graph Cut Minimization

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Abstract. Segmentation of three-dimensional (3D) complicated structures is of great importance for many real applications. In this work we combine *graph cut* minimization method with a variant of the *level set* idea for 3D segmentation based on the *Mumford-Shah* model. Compared with the traditional approach for solving the Euler-Lagrange equation we do not need to solve any partial differential equations. Instead, the *minimum cut* on a special designed graph need to be computed. The method is tested on data with complicated structures. It is rather stable with respect to initial value and the algorithm is nearly parameter free. Experiments show that it can solve large problems much faster than traditional approaches.

AMS subject classifications: 65K10, 49K20, 49K35

Key words: Piecewise constant level set method, energy minimization, graph cut, segmentation, three-dimensional.

1. Introduction

The object of this paper is to extend and test the two-dimensional (2D) graph cut algorithm proposed in [2] for 3D image segmentation. The algorithm is used for minimizing the *piecewise constant level set method* (PCLSM) [16], which is a region based segmentation approach in which object boundaries are detected both with and without gradient information. The PCLSM is minimized by finding the minimum cut on a special graph, thus we need not solve any partial differential equations. This yields an accurate solver, which detects complicated structures. The approach is fast, not very sensitive with respect to initial value and is almost parameter free.

The *level set method* [18] represents evolving interfaces by embedding them in a higher dimensional function ϕ, referred to as the level set function. Traditionally the evolving

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interfaces are represented by the zero level set of ϕ, which is usually given as a signed distance function, and evolves under the influence of forces in the normal direction of ϕ itself. This allows for representing complicated structures which do not depend on the discretization and automatic handling of topological changes in the evolving front, such as merging and splitting, naturally. The level set method is traditionally solved by first deriving an *Euler-Lagrange* equation and then solving the problem using standard numerical schemes. Since its introduction, the method has been widely studied within different fields. It has been applied for image processing problems such as noise removal and segmentation [6,7,16,19,24].

For region based image segmentation, the *Chan-Vese model* [7] used the level set framework to solve the *Mumford-Shah model* [17] for two-phase problems. It was later extended for segmenting multiphase problems [27] by representing the evolving fronts using several ϕ's. The PCLSM [16] was later introduced as an alternative region based image segmentation approach for multiphase problems. It segments images using a single piecewise constant level set function (PCLSF) ϕ. The method has previously been used for image segmentation by among others [8,15,16,21,23,25,26]. Traditionally the PCLSM is solved in a similar fashion as standard level set approaches, by deriving and solving an Euler-Lagrange equation. This is a time consuming procedure, in particular for larger data-sets. Recently a new method for solving the PCLSM was introduced in [2] for 2D segmentation problems. They employ *graph cut* for solving the problem, which is a graph based approach for fast and accurate minimizations of energy functions. Graph cut problems are solved by finding the *minimum cut* on a graph G, which is equivalent to finding the *maximum flow* on G due to the duality theorem of [11]. It was shown in [13] that the minimum cut/maximum flow algorithms can be used for minimizing certain energy functions in computer vision. Graph cut has previously been used for solving variational problems by among others [3, 5, 9, 10, 12]. In this article we extend the 2D approach of [2] for solving 3D multiphase problems, and demonstrate it for segmentation of real *computed tomography* (CT) data and on synthetic *magnetic resonance* (MR) images of the human brain.

An advantage using the level set method is that the evolving interface is implicitly represented by a higher dimensional function ϕ. Complicated structures are therefore naturally handled, as well as topological changes such as merging and breaking. The level set function ϕ is often a signed distance function, which has to be reinitialized to ensure a well-posed problem. Using the PCLSM no reinitialization is required, since ϕ is represented by a piecewise constant function [16]. Furthermore, multiphase problems are solved using a single ϕ as opposed to the approach of [27] which use several ϕ's. When the PCLSM is minimized by graph cut, the computation time is reduced significantly compared to the original formulation, which is shown in [2]. The minimization algorithm is robust and nearly parameter free. It remains stable for all test problems. The only user input required is an initial guess of the mean phase values $\mathbf{c}$. No initialization of ϕ is required. Numerical tests have shown that it is sufficient to provide a naive initial guess of $\mathbf{c}$, with distinct values that need not be in the vicinity of the true mean phase values. The original PCLSM formulation is sensitive to perturbations and ill-posed problems could