Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2009.m9005s

Vol. **2**, No. 4, pp. 427-438
November 2009

Adaptive Parameter Selection for Total Variation Image Deconvolution

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Received 13 March 2009; Accepted (in revised version) 11 June 2009

Abstract. In this paper, we propose a discrepancy rule-based method to automatically choose the regularization parameters for total variation image restoration problems. The regularization parameters are adjusted dynamically in each iteration. Numerical results are shown to illustrate the performance of the proposed method.

AMS subject classifications: 65K10, 68U10
Key words: Image restoration, regularization parameter, total variation.

1. Introduction

Digital image deconvolution plays an important part in various areas of applied sciences such as medical and astronomical imaging, and film restoration. The observed image is often degraded by blurring operations and additive noise. The blurring of images is often caused by a relative motion between the camera and the original scene, the defocusing of the lens system, or atmospheric turbulence.

In digital image processing, images are represented by vectors and matrices. In this format, one-dimensional vectors express two-dimensional images. These vectors are formed by stacking the image column by column. Without loss of generality, we assume that the size of the image is $n \times n$, but all discussions can be applied to images of size $n \times m$. Hence the original and observed images f_{true} and g are expressed by the $n^2 \times 1$ vectors $\boldsymbol{f}_{true}$ and $\boldsymbol{g}$ respectively, and their relationship can be expressed as follows

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f}_{true} + \boldsymbol{n}.$$

Here $\boldsymbol{H}$ is a blurring matrix and $\boldsymbol{n}$ is a vector of zero-mean Gaussian white noise with variance σ^2. The main aim of image deconvolution is to recover the image $\boldsymbol{f}$ from the observed image $\boldsymbol{g}$ such that $\boldsymbol{f} \approx \boldsymbol{f}_{true}$.

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The challenge in image restoration is that it is an ill-posed problem. The simple approach of performing the inverse transformation to the observed image is not feasible since either there doesn't exist an inverse transformation or the inverse transformation is very ill-conditioned; a small perturbation in the observed image can produce a large perturbation in the restored image.

Regularization theory is often used to handle such ill-conditioned problems. One usual approach is to determine the restored image by minimizing the following energy functional

$$\min_{f} \left\| \boldsymbol{H}\boldsymbol{f} - \boldsymbol{g} \right\|_2^2 + \beta \mathscr{R}(\boldsymbol{f}), \tag{1.1}$$

where β is called the regularization parameter and $\mathscr{R}$ is the regularization term. Numerous expressions for $\mathscr{R}$ have been used in literature, such as Tikhonov regularization [12, 24], Total variation (TV) regularization [22], Wavelet regularization [4, 9, 11], *etc*. The energy functional is a weighted sum of the two terms. The first term is the data fitting term and the second term is the regularization term which contains some prior information about the original image to alleviate the problem of ill-conditioning. By adjusting the regularization parameter, a compromise is achieved to suppress the noise and preserve the nature of the original image. Usually, the regularization parameter β is determined by trial-and-error method, the generalized cross validation method [12,13], discrepancy principle [17] or the L-curve method [14]. Also, the regularization parameter can be regarded as the Lagrange multiplier of the constrained minimization problem [3]

$$\min_{f} \mathscr{R}(\boldsymbol{f})$$

subject to

$$\left\| \boldsymbol{H}\boldsymbol{f} - \boldsymbol{g} \right\|_2^2 = \mathrm{E}[\|\boldsymbol{n}\|_2^2] = \sigma^2 n^2, \tag{1.2}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator. The variance σ^2 of the noise can be estimated using the median rule [16].

Applying deblurring and denoising independently is a relatively prevalent concept. Its success is due to the facts that the methods are easy to implement, and solving large linear systems is avoided. In [18, 21, 26], the authors proposed a two-step approach to recover the image when a pilot image $\boldsymbol{u}_{pilot}$ is available. This approach can be formulated as the following consecutive minimization problem:

$$\begin{aligned}
\mathscr{S}_\alpha(\boldsymbol{u}_{pilot}) &= \mathrm{argmin}_f \left\| \boldsymbol{H}\boldsymbol{f} - \boldsymbol{g} \right\|_2^2 + \alpha \left\| \boldsymbol{R}\left(\boldsymbol{f} - \boldsymbol{u}_{pilot}\right) \right\|_2^2, \\
\mathscr{T}_\beta(\mathscr{S}_\alpha(\boldsymbol{u}_{pilot})) &= \mathrm{argmin}_u \frac{1}{2} \left\| \mathscr{S}_\alpha(\boldsymbol{u}_{pilot}) - \boldsymbol{u} \right\|_2^2 + \beta \mathscr{R}(\boldsymbol{u}).
\end{aligned}$$

where $\boldsymbol{R}$ is the regularization matrix, α and β are a regularization parameters. Usually, $\boldsymbol{R}$ is the identity matrix, in which a minimum residual on $\boldsymbol{f}$ subject to a noise constraint is sought, or $\boldsymbol{R}$ is the finite difference matrix, in which the smoothness of the restored image is enhanced. The pilot image can be set to $\boldsymbol{u}_{pilot} = 0$ or the restored image obtained by other methods.