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Simulating Three-Dimensional Free Surface Viscoelastic Flows using Moving Finite Difference Schemes

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Abstract. An efficient finite difference framework based on moving meshes methods is developed for the three-dimensional free surface viscoelastic flows. The basic model equations are based on the incompressible Navier-Stokes equations and the Oldroyd-B constitutive model for viscoelastic flows is adopted. A logical domain semi-Lagrangian scheme is designed for moving-mesh solution interpolation and convection. Numerical results show that harmonic map based moving mesh methods can achieve better accuracy for viscoelastic flows with free boundaries while using much less memory and computational time compared to the uniform mesh simulations.

AMS subject classifications: 65M20, 65N22

Key words: Moving mesh, free surface, viscoelastic flow, solution interpolation.

1. Introduction

Modeling and simulating viscoelastic flows with free boundary have been challenging due to the fact that the constitutive equation adds more complexity to the original Navier-Stokes equation and the moving boundary in free surface flow often requires high resolution meshes to achieve good computational results.

Several computational techniques have been developed for moving interface problems, including volume-of-fluid methods [13], level set methods [20, 21, 25] and diffuse-interface methods [1]. There are also a large number of modifications and hybrid techniques proposed by different people. For example, [26] combines some of the advantages of the volume-of-fluid method with the level set method to obtain a method which is generally superior to either method alone and [10] improved the mass conservation properties

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of the level set method by using Lagrangian marker particles to rebuild the level set in regions which are under-resolved. These techniques were successfully applied to multi-phase or free surface flows (see e.g. [15, 23, 27, 33]). The main challenge is that very fine computational resolution is needed for resolving thin interfaces. Therefore it is practical to implement these methods on adaptive meshes since the problem scale grows rapidly especially in 3D cases. In past years, many adaptive mesh techniques have been proposed which can be classified as adaptive mesh refinement methods and adaptive mesh redistribution methods, see [11,19,30,34]. Using adaptive mesh methods for moving interface problems is also straightforward. For example, [34] simulated two-phase viscoelastic flows using phase-field model with local refined meshes and [8] simulated incompressible two-phase flows using level set method with adaptively redistributed meshes.

In this work, we simulated free surface viscoelastic flows using a moving mesh method (i.e., adaptive mesh redistribution method in the sense of [19]). In particular, we will use the moving mesh algorithms developed in Li *et al.* [16, 17] which redistribute mesh nodes based on harmonic mappings. The moving mesh method based on harmonic mapping has been applied successfully to several complex problems including incompressible flow [6–8], reaction-diffusion systems [22], and dendritic growth [14, 31, 32]. The goal of the moving mesh method is to reduce the computational cost and to enhance the accuracy in resolving the moving interfaces. We designed a moving finite difference based framework which is much faster.

We use incompressible Navier-Stokes equations coupled with Oldroyd-B constitutive equation as the basic models. Phase-field model is used for two-phase flows and level set method is used for free surface flows. We follow Chorin's projection method [4] to keep the velocity field divergence free. For free surface viscoelastic flows, we also split the momentum equation into several sub-equations including convection, diffusion and stress integration. Courant et al. [5] proposed a simple method based on characteristics for discretizing advection equations. These semi-Lagrangian type schemes are popular in many areas because they can be made unconditionally stable. For example, when we use upwind schemes in the level set convection, negative values may become positive near boundaries where velocities point inward if the CFL condition does not hold. But the semi-Lagrangian schemes will not change the sign. In our moving finite difference framework, we adopt the simplest semi-Lagrangian scheme which traces back a straight line characteristic and uses trilinear interpolation to estimate the data, and thus is first-order accurate in both space and time. The main difference is that this scheme is performed on the logical domain and the velocity field is transformed from the physical domain to the logical domain. Although higher order schemes (e.g., BFECC [9] with semi-Lagrangian building blocks [24]) can be implemented on moving meshes, additional storage and complexity related to Jacobian transformations may become problems. Numerical experiments show that the first-order semi-Lagrangian scheme works quite well on moving meshes, which is consistent with the fact that the moving mesh has the ability to redistribute the errors according to the regularity of the solutions. The diffusion and projection will lead to two Poisson equations which are solved using preconditioned conjugate gradient (PCG) method with Jacobi preconditioner. Here a symmetric discretization is used for free surface conditions in order to use