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Mixed Spectral and Pseudospectral Methods for a Nonlinear Strongly Damped Wave Equation in an Exterior Domain

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Abstract. The aim of this paper is to develop the mixed spectral and pseudospectral methods for nonlinear problems outside a disc, using Fourier and generalized Laguerre functions. As an example, we consider a nonlinear strongly damped wave equation. The mixed spectral and pseudospectral schemes are proposed. The convergence is proved. Numerical results demonstrate the efficiency of this approach.

AMS subject classifications: 65N35, 41A05, 33C45, 35L05

Key words: Mixed spectral and pseudospectral methods, exterior problems, strongly damped wave equation.

1. Introduction

We consider the initial and boundary value problem of a nonlinear strongly damped wave equation outside a disc, with smooth boundary conditions,

$$\begin{cases} \partial_t^2 w(\rho,\theta,t) - \partial_t \Delta w - \Delta w + \varphi(w) = F(\rho,\theta,t), & \rho > 1,\ \theta \in \bar{I},\ t \in (0,T], \\ w(\rho,\theta+2\pi,t) = w(\rho,\theta,t), & \rho > 1,\ \theta \in \bar{I},\ t \in [0,T], \\ w(\rho,\theta,0) = w_0(\rho,\theta), \quad \partial_t w(\rho,\theta,0) = w_1(\rho,\theta), & \rho \geq 1,\ \theta \in \bar{I}, \\ w(1,\theta,t) = 0, \qquad \lim\limits_{\rho\to\infty} \rho^{\frac{3}{2}} w(\rho,\theta,t) = 0, & \theta \in \bar{I},\ t \in [0,T], \end{cases} \tag{1.1}$$

where $I = (0, 2\pi)$ and the Laplacian:

$$\Delta w(\rho,\theta,t) = \frac{\partial^2 w(\rho,\theta,t)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial w(\rho,\theta,t)}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 w(\rho,\theta,t)}{\partial \theta^2}.$$

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As pointed out in [15], the above model describes the transversal vibrations of a homogeneous string and the longitudinal vibrations of a homogeneous bar, respectively, subject to viscous effects. The term $-\Delta\partial_t U$ indicates that the stress is proportional not only to the strain, as with the Hooke law, but also to the strain rate as in a linearized Kelvin-Voigt material. Ikehata [11] presented some uniform energy decay estimates of solutions to the linear wave equations with strong dissipation in the exterior domain case.

In this paper, we shall focus on developing the mixed spectral and pseudospectral methods for numerical simulation of problem (1.1), by using Fourier and generalized Laguerre function. Let

$$\mathscr{L}_l^{(\alpha,\beta)}(x)=\frac{1}{l!}x^{-\alpha}e^{\beta x}\partial_x^l(x^{l+\alpha}e^{-\beta x}),\qquad \alpha>-1,\quad \beta>0,\quad l=0,1,\cdots,$$

be the generalized Laguerre polynomials, which are mutually orthogonal in $(0,\infty)$, associated with the weight function $x^\alpha e^{-\beta x}$. The generalized Laguerre polynomials have been used extensively for numerical simulations of various problems in unbounded and exterior domains, see [2, 3, 6-9, 12-14, 19].

The generalized Laguerre functions are defined by (cf. [10]):

$$\widetilde{\mathscr{L}}_l^{(\alpha,\beta)}(x)=e^{-\frac{1}{2}\beta x}\mathscr{L}_l^{(\alpha,\beta)}(x)=\frac{1}{l!}x^{-\alpha}e^{\frac{1}{2}\beta x}\partial_x^l(x^{l+\alpha}e^{-\beta x}),\qquad \alpha>-1,\quad \beta>0, \tag{1.2}$$

which are mutually orthogonal with the weight function x^α. The generalized Laguerre functions are very suitable for numerical simulation of various problems in exterior domains. For instance, Wang, Guo and Wu [16] developed a pseudospectral method for symmetric solutions of partial differential equations outside a disc, by using the generalized Laguerre functions (1.2). Meanwhile, Wang, Guo and Zhang [17] proposed a mixed spectral method for three-dimensional exterior problems using spherical harmonic and generalized Laguerre functions. Moreover, Zhang, Wang and Guo [18] also presented the mixed spectral and pseudospectral methods for linear problems outside a disc, using Fourier and generalized Laguerre functions. But, in practice, it is more interesting and more challenging to consider nonlinear exterior problems. The aim of this paper is to develop the mixed spectral and pseudospectral methods for two-dimensional nonlinear exterior problem (1.1), by using Fourier and generalized Laguerre function.

This paper is organized as follows. In Section 2, we recall some basic results of the mixed approximation using Fourier and generalized Laguerre functions. In Section 3, we propose the mixed spectral and pseudospectral methods for problem (1.1). In Section 4, we also present some numerical results demonstrating the high efficiency of these methods. The final section is for concluding remarks.