Numer. Math. Theor. Meth. Appl. Vol. **4**, No. 3, pp. 335-358
doi: 10.4208/nmtma.2011.m1021 August 2011

Analysis of Direct and Inverse Cavity Scattering Problems

Gang Bao[1,2,*], Jinglu Gao[3] and Peijun Li[4]

[1] *Department of Mathematics, Zhejiang University, Hangzhou 310027, China.*
[2] *Department of Mathematics, Michigan State University, East Lansing, MI 48824, USA.*
[3] *School of Mathematical Sciences, Jilin University, Changchun 130023, China.*
[4] *Department of Mathematics, Purdue University, West Lafayette, IN 47907, USA.*

Received 21 July 2010; Accepted (in revised version) 11 November 2010

Available online 20 June 2011

Abstract. Consider the scattering of a time-harmonic electromagnetic plane wave by an arbitrarily shaped and filled cavity embedded in a perfect electrically conducting infinite ground plane. A method of symmetric coupling of finite element and boundary integral equations is presented for the solutions of electromagnetic scattering in both transverse electric and magnetic polarization cases. Given the incident field, the direct problem is to determine the field distribution from the known shape of the cavity; while the inverse problem is to determine the shape of the cavity from the measurement of the field on an artificial boundary enclosing the cavity. In this paper, both the direct and inverse scattering problems are discussed based on a symmetric coupling method. Variational formulations for the direct scattering problem are presented, existence and uniqueness of weak solutions are studied, and the domain derivatives of the field with respect to the cavity shape are derived. Uniqueness and local stability results are established in terms of the inverse problem.

AMS subject classifications: 78A46, 78M30

Key words: Electromagnetic cavity, direct problem, inverse problem, finite element methods, boundary integral equations.

1. Introduction

The radar cross section (RCS) is a measure of the detectability of a target by radar system. Deliberate control in the form of enhancement or reduction of the RCS of a target is of no less importance than many radar applications. The cavity RCS caused by jet engine inlet ducts or cavity-backed antennas can dominate the total RCS. A thorough understanding of

*Corresponding author. *Email addresses:* `bao@math.msu.edu` (G. Bao), `jinglugao@gmail.com` (J. Gao), `lipeijun@math.purdue.edu` (P. Li)

the electromagnetic scattering characteristic of a target, particularly a cavity, is necessary for successful implementation of any desired control of its RCS, and is of high interest to the scientific and engineering community.

Time-harmonic analysis of cavity-backed apertures with penetrable material filling the cavity interior has been examined by numerous researchers in the engineering community, such as Jin [18], Jin and Volakis [19], Liu and Jin [21], Wood and Wood [30], and references cited therein. Mathematical treatment of the direct scattering problems involving cavities can be found in Ammari *et al* [2,3], Bao and Sun [7], Van and Wood [28], where a non-local transparent boundary condition, based on the Fourier transform, is proposed on the open aperture of the cavity. It is a common assumption that the cavity opening coincides with the aperture on an infinite ground plane, and hence simplifying the modeling of the exterior (to the cavity) domain. This limits the application of these methods since many cavity openings are not planar. Recently, Wood [29] has developed a technique that is capable of characterizing the scattering by over-filled cavities in the frequency domain, where an artificial boundary condition, based on Fourier series, is introduced on a semicircle enclosing the cavity. The solution domain is the cavity plus the interior region enclosed by the semicircle, which may be large and thus computationally demanding if the aperture of the cavity is wide. This paper aims to develop an efficient alternative for dealing with both regular and over-filled cavities with arbitrary shape, and analyze the associated direct and inverse electromagnetic scattering problems.

Specifically, we consider a time-harmonic electromagnetic plane wave incident on an open cavity embedded in an infinite ground plane. The ground plane and the wall of the cavity are perfect electric conductors, and the open cavity is filled with a nonmagnetic material which may be inhomogeneous. The infinite upper half-space above the ground plane and the cavity is composed of a homogeneous medium characterized by its permittivity ε_0 and permeability μ_0. Two fundamental polarizations, transverse electric (TE) and transverse magnetic (TM), are considered for the direct and inverse electromagnetic scattering from the cavity.

Given a time-harmonic plane incident wave and the shape of the cavity, the direct scattering problem is to predict the field distribution away from the cavity. We present a method of symmetric coupling of finite element and boundary integral equations. Computationally, the symmetric coupling leads to a complex symmetric coefficient matrix which can be efficiently stored and solved, especially for three-dimensional problems. In this method, the unbounded region is first divided into an interior region and an exterior region through an artificial boundary. The field in the interior region is formulated using the finite element method, and the field in the exterior region is formulated via the boundary integral method. The interior and exterior fields are subsequently coupled by the continuity conditions at the boundary separating the two regions. Therefore, the boundary integral equation essentially provides a transparent boundary condition on the boundary of the truncated domain to avoid artificial wave reflection. The position of the artificial boundary is rather flexible and can chosen to greatly reduce the computational effort: it will just be the aperture on the ground plane for a regular cavity; it can be put as close as possible to the opening of the overfilled cavity. In the general two-dimensional setting, we