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Extrapolation of Mixed Finite Element Approximations for the Maxwell Eigenvalue Problem

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Abstract. In this paper, a general method to derive asymptotic error expansion formulas for the mixed finite element approximations of the Maxwell eigenvalue problem is established. Abstract lemmas for the error of the eigenvalue approximations are obtained. Based on the asymptotic error expansion formulas, the Richardson extrapolation method is employed to improve the accuracy of the approximations for the eigenvalues of the Maxwell system from $\mathcal{O}(h^2)$ to $\mathcal{O}(h^4)$ when applying the lowest order Nédélec mixed finite element and a nonconforming mixed finite element. To our best knowledge, this is the first superconvergence result of the Maxwell eigenvalue problem by the extrapolation of the mixed finite element approximation. Numerical experiments are provided to demonstrate the theoretical results.

AMS subject classifications: 65M10, 65N30

Key words: Maxwell eigenvalue problem, mixed finite element, asymptotic error expansion, Richardson extrapolation.

1. Introduction

The Maxwell eigenvalue problems are of basic importance in designing the microwave resonators, microwave ovens, and communication equipments [2,3,16]. Assuming that the electromagnetic fields are time-harmonic, after elimination of the magnetic field intensity, the Maxwell eigenvalue problem can be formulated by [5,11]

$$\nabla \times \nabla \times p = \lambda p, \qquad \text{in } \Omega, \tag{1.1}$$

$$\nabla \cdot p = 0, \qquad \text{in } \Omega, \tag{1.2}$$

$$p \times n = 0, \qquad \text{on } \partial\Omega, \tag{1.3}$$

where Ω is a bounded cubic domain in $\mathscr{R}^3$.

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There are many results for the Maxwell eigenvalue problems based on the finite element modeling and analysis, see, e.g., [1,3,5,8,15]. It is known, see [19], that the modeling of electromagnetic resonances is delicate. The early attempts to calculate FEMs approximation may lead to the occurrence of non-physical, so-called spurious eigenmodes [4,16]. In order to overcome this difficulty, many methods have been proposed. In general, there are two possibilities: either one imposes the constraint of divergence-freeness on the problem, or one looks for an easy identification of the eigenvectors from the kernel of the curl operator. In order to impose the divergence-free constraint on the problem, one may try to incorporate this property in the definition of the discrete function spaces [7, 11, 30, 31, 37]. Several researchers prefer to impose this constraint implicitly, using mixed formulations [2, 3, 15, 20]. The finite element approximation of the Maxwell equation has also been studied to solve the electromagnetic field in [11,21,28].

When approximating the eigenvalue of Maxwell equations by the finite element method, it has been proved that it is not easy to distinguish the spurious values [4] using the nodal elements, especially for the lower order nodal elements. Such spurious values can be removed by the weighted regularization method [10] or by the least-square method [6]. For the non-smoothing solution not in $H^1(\Omega)$, a element-local L_2 projection technique was presented in [14] to deal with the nonconvex Lipschitz polyhedron with reentrant corners and edges. However, it has been shown that edge elements for the eigenvalue Maxwell equations can have a good approximation on the affine mesh [2–4], though can not achieve optimal approximation on non-affine mesh for the lower order edge elements [5]. In [23,24], one of the variational form EQ_1^{rot} of Rannacher-Turek nonconforming element was proposed and numerical examples were shown that it can produce better approximations for the eigenvalue of elliptic problem. In three dimensions, this nonconforming finite element is a face-element. And it has been applied to solve the time-harmonic Maxwell's equations with the absorbing boundary condition by Douglag etc. in [12].

To enhance the finite element eigenvalues approximation, one of the most important techniques is the polynomial preserving recovery (PPR) [29, 36]. Remarkable fourth order convergence is observed for linear elements under structured meshes as well as unstructured initial meshes with the conventional refinement. The extrapolation method is another important technique to enhance the finite element eigenvalue approximation. For many eigenvalue problems, the extrapolation method can be applied to improve the accuracy of eigenvalue approximation on the structured mesh. Based on asymptotic expansions of error approximation, one or two orders of convergence rate can be improved by a simple extrapolation postprocessing, see, e.g., [9,17,18,22,25,35]. These accelerators are all based on an extremely important assumption that the solution should be higher smoothing. Most of these researches were presented for conforming finite elements or conforming mixed finite elements except for [25,35]. For mixed finite element, one of the difficulties is that it is hard to derive asymptotic error expansion formulations. So far, we have not found any superconvergence results for the Maxwell eigenvalue problem by the extrapolation of the mixed finite element approximation.

In this paper, we assume that the exact solution is sufficiently smooth in order to employ the extrapolation technique. For the lowest order Nédélec mixed finite element and