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Coupled Models and Parallel Simulations for Three-Dimensional Full-Stokes Ice Sheet Modeling

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Abstract. A three-dimensional full-Stokes computational model is considered for determining the dynamics, temperature, and thickness of ice sheets. The governing thermo-mechanical equations consist of the three-dimensional full-Stokes system with nonlinear rheology for the momentum, an advective-diffusion energy equation for temperature evolution, and a mass conservation equation for ice-thickness changes. Here, we discuss the variable resolution meshes, the finite element discretizations, and the parallel algorithms employed by the model components. The solvers are integrated through a well-designed coupler for the exchange of parametric data between components. The discretization utilizes high-quality, variable-resolution centroidal Voronoi Delaunay triangulation meshing and existing parallel solvers. We demonstrate the gridding technology, discretization schemes, and the efficiency and scalability of the parallel solvers through computational experiments using both simplified geometries arising from benchmark test problems and a realistic Greenland ice sheet geometry.

AMS subject classifications: 65M60, 65M55, 65Y05, 65Z05, 68U20, 68W10

Key words: Ice sheet modeling, nonlinear Stokes equation, finite element method, parallel implementation, centroial Voronoi Delaunay meshes.

1. Introduction

The computational modeling of glaciers and ice sheets has been a subject of growing interest because of the influential role they play in global sea level and climate change studies

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[6–8, 13]. Among the different types of approaches employed, the full three-dimensional Stokes ice sheet model is generally accepted to truly model ice sheet flows [18, 28, 41, 42]. A three-dimensional full-Stokes ice sheet computational model requires the integration of effective gridding strategies, discretization schemes, couplers for data exchange between model components, and efficient, scalable solvers.

Finite difference, finite volume, and finite element methods have proven successful for high-resolution computational simulations based on the full-Stokes model with free surface evolution [19, 40, 42]. However, the applicability of the methods for large-scale, high-resolution simulations of realistic glaciers and ice sheets remains an open question. Our approach uses finite element discretizations of the mechanical and thermal components. Based on observations from field and satellite-based studies [5, 34, 35, 37], another key component is a high-quality, adaptive, variable resolution meshing scheme that can often significantly reduce the computational cost while maintaining comparable solution accuracy relative to quasi-uniform grids. Recently, centroidal Voronoi tessellation (CVT) based mesh generation techniques have been widely incorporated into finite volume and finite element approximation schemes for convection-diffusion equations [12, 22], the Navier-Stokes equations [23], and the shallow water equations [43]; CVT-based mesh generation offers significant advantages, compared to other meshing algorithms, for improving discretization/solution accuracy and controlling local mesh sizes [11, 17, 21, 22, 24, 43].

This paper reports on progress made towards the development of an efficient, parallel, finite element solver for three-dimensional, full-Stokes ice sheet modeling. The coupled thermo-mechanical processes are numerically approximated by a scalable system using Message Passing Interface (MPI) that is computing intensive and capable of performing large amounts of data transfer during the simulation processes. We first present the governing equations, the methods we use for their discretization, and the parallel solvers we use. We then validate our numerical schemes and parallel implementations by applying them to benchmark tests having simple geometries put forward by the ice sheet modeling community. The feasibility of our parallel solvers is then demonstrated on realistic geometries by applying our package to idealized simulations of the Greenland ice sheet.

We consider the evolution of momentum, temperature, and thickness of an ice sheet having ice-atmosphere and ice-bedrock boundaries only. Moreover, we consider a simplified set of boundary conditions along those boundaries. These simplifications are in concert with the main goals of this paper, which are as follows:

- develop and test a finite element discretization of the full, three-dimensional Stokes model for ice sheet dynamics, coupled to an energy equation for the evolution of the temperature in the ice sheet and a conservation equation for the evolution of the ice sheet thickness;
- implement high-quality, variable-resolution prismatic grids based on centroidal Voronoi Delaunay triangulations;
- develop and test efficient parallel solvers for the discretized coupled system;
- demonstrate that the combined components result in a potentially powerful tool for ice sheet modeling.