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A Regularization Semismooth Newton Method for P_0-NCPs with a Non-monotone Line Search

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Abstract. In this paper, we propose a regularized version of the generalized NCP-function proposed by Hu, Huang and Chen [J. Comput. Appl. Math., 230 (2009), pp. 69–82]. Based on this regularized function, we propose a semismooth Newton method for solving nonlinear complementarity problems, where a non-monotone line search scheme is used. In particular, we show that the proposed non-monotone method is globally and locally superlinearly convergent under suitable assumptions. We test the proposed method by solving the test problems from MCPLIB. Numerical experiments indicate that this algorithm has better numerical performance in the case of $p = 5$ and $\theta \in [0.25, 075]$ than other cases.

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1. Introduction

The nonlinear complementarity problem (NCP for short) is to find a point $x \in \mathfrak{R}^n$ such that

$$x \geq 0, \quad f(x) \geq 0, \quad x^T f(x) = 0, \tag{1.1}$$

where $f : \mathfrak{R}^n \to \mathfrak{R}^n$ is a continuously differentiable mapping with $f := (f_1, f_2, \cdots, f_n)^T$. If f is a P_0-function, i.e.,

$$\max_{1 \leq i \leq n \; x_i \neq y_i} (x_i - y_i)(f_i(x) - f_i(y)) \geq 0$$

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holds for all $x, y \in \Re^n$ and $x \neq y$, then we call (1.1) the P_0-NCP. The NCP has various applications in operation research, economics, and engineering (see, for example, [1–4]). Various methods for solving the NCP have been proposed in the literature (see, for example, [5–18]). In all the mentioned methods, the so-called NCP-function, i.e., $\phi(a,b)=0$ if and only if $a \geq 0, b \geq 0, ab=0$, plays an important role.

Recently, a family of new NCP-functions was proposed in [19], which is defined by

$$\omega_{\theta p}(a,b) := \sqrt[p]{\theta(|a|^p+|b|^p)+(1-\theta)|a-b|^p} - a - b, \tag{1.2}$$

where $p \in (1,+\infty), \theta \in (0,1]$, and $(a,b) \in \Re^2$. When $\theta = 1$, the function $\omega_{\theta p}$ reduces to the function in [20, 21], and when $\theta = 1$ and $p = 2$, the function $\omega_{\theta p}$ reduces to the Fischer-Burmeister function [10]. It was showed in [19, Propositions 2.1 and 2.3] that the function $\omega_{\theta p}(\cdot,\cdot)$ is an NCP function and a semismooth function in $\Re^2$. Moreover, it is known that $\omega^2_{\theta p}(\cdot,\cdot)$ is continuously differentiable and strongly semismooth in $\Re^2$ (see, e.g., [19, Proposition 2.5]).

In this paper, by using the symmetrically perturbed technique proposed in [22, 23], we give a regularized version of the generalized NCP-function (1.2), which is defined by

$$\begin{aligned} &\phi_{\theta p}(\mu,a,b) \\ &= \sqrt[p]{\theta(|\mu a+b|^p+|a+\mu b|^p)+(1-\theta)|\mu a+b-(a+\mu b)|^p} - ((\mu a+b)+(a+\mu b)) \\ &= \sqrt[p]{\theta(|\mu a+b|^p+|a+\mu b|^p)+(1-\theta)|(1-\mu)(a-b)|^p} - (1+\mu)(a+b), \end{aligned} \tag{1.3}$$

where $(\mu,a,b) \in \Re_+ \times \Re \times \Re$; and $\theta \in [0,1]$ and $p \in (1,+\infty)$ are two given parameters. It is obvious that $\phi_{\theta p}(0,\cdot,\cdot)$ is an NCP function. For all $z := (\mu, x) \in \Re_+ \times \Re^n$, we define

$$H_{\theta p}(z) := \begin{pmatrix} \mu \\ \Phi_{\theta p}(z) \end{pmatrix} \qquad \Psi_{\theta p}(z) := \|H_{\theta p}(z)\|^2 = \mu^2 + \|\Phi_{\theta p}(z)\|^2, \tag{1.4}$$

where

$$\Phi_{\theta p}(z) := \begin{pmatrix} \phi_{\theta p}(\mu, x_1, f_1(x)) \\ \vdots \\ \phi_{\theta p}(\mu, x_n, f_n(x)) \end{pmatrix}. \tag{1.5}$$

It is easy to see that $z := (\mu, x)$ is a solution of $H_{\theta p}(z) = 0$ if and only if $\mu = 0$ and x solves the NCP (1.1). We will show that the function $H_{\theta p}(z)$ defined in (1.4) is coercive with respect to z. Such a property can improve the global convergence of the semismooth Newton method (see [24, 25]; also see [26] for comparisons about the conditions of the global convergence). It should be noted that the function $H_{\theta p}(z)$ defined in (1.4) is not coercive with respect to z if the function $\phi_{\theta p}(\cdot,\cdot,\cdot)$ in the definition of $H_{\theta p}(\cdot)$ is replaced by the function $\omega_{\theta p}(\cdot,\cdot)$ given in (1.2).

Many numerical methods based on the NCP-function (or the smoothed NCP-function) have not only good convergence, but also good numerical results, such as the semismooth