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The Bases of the Non-Uniform Cubic Spline Space $S_3^{1,2}(\Delta_{mn}^{(2)})$

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Abstract. In this paper, the dimension of the nonuniform bivariate spline space $S_3^{1,2}(\Delta_{mn}^{(2)})$ is discussed based on the theory of multivariate spline space. Moreover, by means of the Conformality of Smoothing Cofactor Method, the basis of $S_3^{1,2}(\Delta_{mn}^{(2)})$ composed of two sets of splines are worked out in the form of the values at ten domain points in each triangular cell, both of which possess distinct local supports. Furthermore, the explicit coefficients in terms of B-net are obtained for the two sets of splines respectively.

AMS subject classifications: 65D07

Key words: Bivariate spline, conformality of smoothing cofactor method, B-net, nonuniform type-2 triangulation.

1. Introduction

In the past years a vast amount of work has been done on the subject of multivariate approximation that is nowadays an increasingly active research area, see, e.g., [7]. The field is both fascinating and intellectually stimulating since much classical univariate theory does not straightforwardly generalize to the multivariate one. As a result, new tools have been developed, such as wavelets, multivariate splines [22, 23], radial-basis functions [2] and so on. Among these many developments are results on multivariate splines which are applied widely in approximation theory, computer aided geometric design and finite element method.

As known, the nonuniform rational B-splines (NURBS) scheme has become a de facto standard in Computer Aided Geometric Design (CAGD), which is based on algebraic polynomials [9, 15, 18, 20]. It is a powerful tool for constructing free-form curves and surfaces. Recently, some new alternatives to the rational model have been proposed for constructing fair-shape-preserving approximations that inherit major geometric properties such as positivity, monotonicity and convexity [16, 21], whose limiting cases are B-splines. However, both B-spline surfaces and the new alternatives are constructed in the form of tensor

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product, which implies that the degrees of the surfaces are the multiplication of that of the parameters in two directions. Take bicubic B-spline surface for in stance, it is a surface of degree six. As a result, due to the high degree, there may be some inflection points on the surface. Moreover, the bivariate function can not reproduce any polynomial of nearly best degree. To keep away the shortcomings, we are now going to directly generalize the bivariate splines and compute the bases by using the Conformality of Smoothing Cofactor Method [23] in this paper and then make a further study of cubic spline quasi-interpolation in another one.

As a matter of fact, the Conformality of Smoothing Cofactor Method has played a great role on the dimension of multivariate splines and the computation of the bases in multivariate spline. In much detail, the Global Conformality Condition at the grid points of a partition Δ provides a basic tool in determining the dimension of $S_k^{\mu}(\Delta)$, i.e, the multivariate spline space with degree k and smoothness μ over the domain D with respect to the partition Δ [13,22,23,29]. It is a fundamental problem in theory of multivariate splines to determine the dimension [1,5,6,10,17]. As mentioned in [19], the dimension depends heavily on the geometry of the partition Δ. By means of the Conformality of Smoothing Cofactor Method, important theory on the dimension of $S_k^{\mu}(\Delta)$ has been introduced in next section. Besides this, the Conformality of Smoothing Cofactor Method has played a good role on the computation of the bases of the spline space $S_k^{\mu}(\Delta)$ [27,28]. Recently, Liu, et al. [14] determined the dimension and construct a local support basis of the space $S^{1,d}(\Delta(2))$, for $d = 0$, 1 of the spline functions over the type-2 nonuniform triangulation. The bases of two bivariate cubic and quartic spline spaces on uniform type-2 triangulation $\Delta_{mn}^{(2)}$ were worked out in [12,24], respectively. As an application, spline quasi-interpolation operators have been presented. On uniform type-2 triangulation, spline quasi-interpolation has been investigated thoroughly in [12,24]. On nonuniform type-2 triangulation, a class of quasi-interpolation operators were proposed [3], and their convergence results and error estimates were discussed in [4,11,25,26].

However, the results on spline quasi-interpolation on nonuniform type-2 triangulations in above are almost restricted in the bivariate quadratic B-splines. The reason for it may be the lack of computation of the bases. Since nonuniform triangulation may be more useful than the uniform ones, and bivariate nonuniform splines are important but difficult, we focus on the study of the bases of some nonuniform cubic spline space in this paper. Moreover, we have completed the construction of the corresponding spline quasi-interpolation formula in another paper.

A brief outline of this article is as follows. In Section 2, we discuss the dimension of nonuniform bivariate spline space $S_3^{1,2}(\Delta_{mn}^{(2)})$. Secondly, by using the Conformality of Smoothing Cofactor Method, we work out the basis composed of two sets of splines B_{ij}^1 and B_{ij}^2 with distinct local supports in Section 3, and then discuss their properties. Finally, in section 4, by means of the barycentric coordinates expressions of the two sets of splines, the explicit coefficients in terms of B-net are worked out. As known, the computation of multiple integrals can be converted into the sum of the coefficients in terms of B-net over triangular domain.