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Hierarchical Framework for Shape Correspondence

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Abstract. Detecting similarity between non-rigid shapes is one of the fundamental problems in computer vision. In order to measure the similarity the shapes must first be aligned. As opposite to rigid alignment that can be parameterized using a small number of unknowns representing rotations, reflections and translations, non-rigid alignment is not easily parameterized. Majority of the methods addressing this problem boil down to a minimization of a certain distortion measure. The complexity of a matching process is exponential by nature, but it can be heuristically reduced to a quadratic or even linear for shapes which are smooth two-manifolds. Here we model the shapes using both local and global structures, employ these to construct a quadratic dissimilarity measure, and provide a hierarchical framework for minimizing it to obtain sparse set of corresponding points. These correspondences may serve as an initialization for dense linear correspondence search.

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1. Introduction

Knowing correspondence between shapes is required for various applications, such as shape retrieval, registration, deformation, shape morphing, symmetry, self-similarity detection, etc. Detecting accurate correspondence between non-rigid shapes is a hard problem, since in general it cannot be parameterized by a finite number of unknowns. It can be cast as an assignment problem, and as such is NP-hard. A common approach for detecting correspondence between shapes differing by a certain class of transformations consists of employing shape properties that remain invariant under these transformations. These invariant surface properties are used to formulate a measure of dissimilarity between the shapes. By minimizing it one finds the correct matching. Here we use a matching scheme based on local and global surface properties, namely, local surface descriptors and global metric structures. The proposed method is demonstrated with two different types of

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metrics - geodesic and diffusion, and different surface descriptors that include histograms of geodesic and diffusion distances, heat kernel signatures [39], and related descriptors based on the Laplace-Beltrami operator [12].

The main issue addressed in this paper is the complexity of the matching. Direct comparison of their pointwise surface descriptors and metric structures of two shape given by sampled surfaces is combinatorial in nature (the metric comparison problem was addressed in [25]). The main contribution of this work is a multi-resolution matching algorithm that can handle large number of points, and is able to produce correspondence consistent in terms of both pointwise and pairwise surface properties. According to the proposed scheme, at the lowest resolution the algorithm solves the correspondence problem exactly. The correspondence information is then propagated to a higher resolution, and refined using small neighborhoods of the matched points - thus we effectively reduce the size of the matching problem. The algorithm iterates between correspondence propagation and refinement, until a desired number of matches is found.

The rest of the paper is organized as follows: a brief review of the previous work is presented in the next section. Section 3 presents the correspondence problem formulation, followed by Section 4 where we present the hierarchical framework. In Section 5 we elaborate on distances and descriptors, and in Section 6 we discuss the numerical aspects. Section 7 contains the matching results, and comparison to the state-of-art algorithms, followed by Section 8 that concludes the paper.

2. Previous work

A wide range of methods were suggested in recent years for matching non-rigid shapes. Zigelman *et al.* [44], and Elad and Kimmel [15] suggested a method for matching isometric shapes by embedding them into a Euclidian space using multidimensional scaling (MDS), thus obtaining isometry invariant representations, followed by rigid shape matching in that space. Bronstein *et al.* [5] showed that for some surfaces, such as faces, a spherical domain better captures intrinsic properties. Mapping onto a sphere was also used for cortex alignment in medical imaging, for instance by Glaunès *et. al.* [16], Tosun *et. al.* [40] and Durrleman *et. al.* [13]. Since it is generally impossible to embed a non-flat 2D manifold into a flat Euclidean domain without introducing some errors, the inherited embedding error affects the matching accuracy of all methods of this type. In order to eliminate this embedding error, Memoli and Sapiro [25] introduced the Gromov-Hausdorf distance [8] into the shape matching arena. Soon after, Bronstein *et al.* [6] formulized the Gromov-Hausdorf distance as a solution of a continuous optimization problem, which they called the generalized multi-dimensional scaling (GMDS). It performs a direct embedding between two non-rigid shapes, which does not suffer from the unbounded distortion of an intermediate ambient space. Lipman and Funkhouser [23] used the fact that isometric transformation between two shapes is equivalent to a Möbius transformation between their conformal mappings, and obtained this transformation by comparing the respective conformal factors. However, there is no guarantee that this result minimizes the cumulative difference between geodesic distances measure between pairs of matched points.