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Robust a Simulation for Shallow Flows with Friction on Rough Topography

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Abstract. In this paper, we propose a robust finite volume scheme to numerically solve the shallow water equations on complex rough topography. The major difficulty of this problem is introduced by the stiff friction force term and the wet/dry interface tracking. An analytical integration method is presented for the friction force term to remove the stiffness. In the vicinity of wet/dry interface, the numerical stability can be attained by introducing an empirical parameter, the water depth tolerance, as extensively adopted in literatures. We propose a problem independent formulation for this parameter, which provides a stable scheme and preserves the overall truncation error of $\mathcal{O}(\Delta x^3)$. The method is applied to solve problems with complex rough topography, coupled with h-adaptive mesh techniques to demonstrate its robustness and efficiency.

AMS subject classifications: 65M10, 65M15, 65N30

Key words: Shallow water, free interface, Manning force.

1. Introduction

Shallow water equations (SWEs) have been extensively applied to model hydrodynamic phenomena such as estuary and coastal tidal flows, bore wave propagation, surface irrigation, lake and reservoir hydrodynamics, and open channel flows. Although the numerical methods for shallow water equations have been studied thoroughly in the last decades, there are still some persistent difficulties in their application to practical models, which are often slightly different from the homogeneous shallow water equations. In the case of complex and rough topography, the differences under consideration here are very

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typical, including the friction force term of Manning form [1] due to the topography roughness, and the wet/dry interface due to wave propagating on a dry topography in practical applications.

The Manning friction force turns out to be a stiff source term in case of small water depth in the vicinity of wet/dry interface. As Bradford and Sanders [2] pointed out, the Manning formula requires division by water depth h, which results in an unrealistically large prediction of friction force in shallow regions near wet/dry interface, making the momentum equations stiff and the solution sensitive to water depth. Typical discretized methods have been applied to the friction source term, including the explicit Euler method [3,4], the implicit Euler method [5,6], and the semi-implicit method [7,8]. However, to keep the numerical stability and to control truncation error both impose more severe constraints on time step than the CFL condition. We apply the Strang splitting to separate the Manning friction force term from the shallow water equations, then give the analytical solution in the splitting step when handling the friction source term thus the constraint on time step due to the stiffness is released.

The other problems are the incorrect diffusion and the numerical sensitivity near wet/dry interface. The averaging process of data in a partially wetted cell at the wet/dry interface may give a very small water depth and wet the faces of this cell. As it is averaged on the entire cell instead of the wet part, artificial spreading of water into neighbour dry cells leads to the qualitatively incorrect wet/dry interface diffusing [2]. Furthermore, the small water depth makes the numerical scheme instable that additional error is produced. Many authors have studied the techniques for the wet/dry front. The positivity of the water height and the well-balance property under the presence of dry areas is achieved by two basic ingredients: a positivity reconstruction [9–11] and an additional time step constraint [12, 13]. The basic idea of the latter method is to reduce the time step only for the edges that contribute to the outflow of these cells. We note that the new technique for treating wet/dry fronts in the context of Roe schemes presented in [11,14], which consists of an adequate nonlinear Riemann solver (exactly solved) at the intercells where a wet/dry transition has detected. In this work, we adopt the method in [9]. Precisely, the numerical errors of velocities u and v which are computed by $(hu)/h$ and $(hv)/h$ are amplified in case of $h \to 0$. As a remedy, an artificial parameter is prescribed in the numerical scheme by many authors [2, 4, 15–17], which is referred as *water depth tolerance* and denoted by h_{tol} later on. In case of water depth smaller than h_{tol}, the velocity components are set to be zero and the local fluxes are neglected to suppress possible instability. The numerical results therein illustrate that this technique works if magnitude of the parameter is appropriate. However, the value of the empirical parameter h_{tol} is problem dependent. If h_{tol} is chosen large enough to prevent wet/dry interface diffusion, the overall accuracy may be spoiled by the error on wet/dry interface. On the other hand, as reported in [2] and [16], h_{tol} has been varied by one order of magnitude higher and lower in a variety of frictionless problems with a negligible influence on the resulting solution. But for problems with bed friction of the Manning form, the model becomes even more sensitive to the value of h_{tol}. We remove the sensitivity by directly integrating the friction source term, and propose a problem independent formulation of h_{tol} preserving the overall truncation error of