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On the Approximation of the Derivatives of Spline Quasi-Interpolation in Cubic Spline Space $S_3^{1,2}(\Delta_{mn}^{(2)})$

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Abstract. In this paper, based on the basis composed of two sets of splines with distinct local supports, cubic spline quasi-interpolating operators are reviewed on nonuniform type-2 triangulation. The variation diminishing operator is defined by discrete linear functionals based on a fixed number of triangular mesh-points, which can reproduce any polynomial of nearly best degrees. And by means of the modulus of continuity, the estimation of the operator approximating a real sufficiently smooth function is reviewed as well. Moreover, the derivatives of the nearly optimal variation diminishing operator can approximate that of the real sufficiently smooth function uniformly over quasi-uniform type-2 triangulation. And then the convergence results are worked out.

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1. Introduction

As is known, the nonuniform rational B-splines scheme has become a de facto standard in Computer Aided Geometric Design, which is a powerful tool for constructing free-form curves and surfaces [3, 7, 14, 16]. Due to its rational model, some new alternatives have been proposed for constructing fair-shape-preserving approximations recently [8–10, 15]. However, both B-spline surfaces and the new alternatives are constructed in the form of tensor product, which implies that the degrees of the surfaces are the addition of that of the parameters in two directions so that there may be some inflection points on the surface. Moreover, the bivariate function can not reproduce any polynomial of nearly best degree. Furthermore, it is restricted to construct surfaces over the rectangular mesh. Hence, to

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avoid the shortcomings, it is very important to study multivariate spline functions theoretically. Since multivariate spline functions are heavily dependent on the geometric property of the domain partitions, it is so complex that the non-Cartesian product multivariate spline functions has not been developed radically for a long time. But all is changed until the construction of the Conformality of Smoothing Cofactor Method [17,18].

In a specific way, the smooth cofactor and conformality condition has been introduced to which the polynomials must satisfy by analysing the relation between the polynomials over two adjacent cells [17,18]. The conformality condition establishes the equivalent conversion between multivariate spline functions and the corresponding algebraic problems. As a result, the Conformality of Smoothing Cofactor Method provides an algebraic approach to studying the multivariate spline functions, including the dimension and the locally supported basis functions in multivariate spline spaces [17,18,22,23], etc., which are difficult but important. The dimension of the multivariate spline function space $S_k^{\mu}(\Delta)$ i.e., the multivariate spline space with degree k and smoothness μ over the domain D with respect to the partition Δ have been widely developed in [4,13,14,17,18,24]. Recently, Liu, Hong, and Cao [6] determined the dimension and construct a local support basis of the space $S^{1,d}(\Delta(2))$, for $d = 0,1$ of the spline functions over the type-2 nonuniform triangulation. The basis functions of bivariate cubic and quartic spline spaces on uniform type-2 triangulation have been derived in [5,19], respectively, where spline quasi-interpolation has been also investigated thoroughly. These spline quasi-interpolating operators can reproduce any polynomial of (nearly) best degrees, respectively. Moreover, spline quasi-interpolation defined by discrete linear functionals based on a fixed number of triangular mesh-points has been investigated, which showed that they could approximate a real function and its partial derivatives up to an optimal order in [1,2].

However, in view of the complexity in computation of the bases, the study on spline quasi-interpolation over nonuniform type-2 triangulations are almost restricted in bivariate quadratic B-splines, see [20,21]. Since multivariate approximation over irregular triangulations may be more important than that over uniform triangulations, we have computed the cubic splines in [11], and have constructed the cubic spline quasi-interpolation in [12] by using the Conformality of Smoothing Cofactor Method [17,18]. Now we shall make a further study on the approximation of the derivatives of the cubic spline quasi-interpolation in this paper.

A brief outline of this article is organized as follows. In Section 2, we review the dimension and the bases in $S_3^{1,2}(\Delta_{mn}^{(2)})$. Based on five mesh points or the center of the support of each spline B_{ij}^1 and five mesh points of the support of each spline B_{ij}^2, the representation of spline quasi-interpolation is investigated, which can reproduce any polynomial in $\mathbb{P}_2 \cup \{x^2y, xy^2\}$. Then in section 3, we make a further study of the derivatives of the cubic spline quasi-interpolation, which can approximate the derivatives of the real sufficiently smooth function uniformly over quasi-uniform triangulation.

2. Review of representation of spline quasi-interpolation in $S_3^{1,2}(\Delta_{mn}^{(2)})$

The domain $\Omega = [a,b]\times[c,d]$ is partitioned into mn rectangular cells $\Omega_{ij} = [x_i, x_{i+1}]\times[y_j, y_{j+1}]$, $i = 0,\cdots,m-1$ and $j = 0,\cdots,n-1$, where m,n are given positive integers,