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Generalized and Unified Families of Interpolating Subdivision Schemes

Ghulam Mustafa[1,*], Pakeeza Ashraf[2] and Jiansong Deng[2]

[1] *Department of Mathematics, The Islamia University of Bahawalpur, Pakistan.*
[2] *University of Science and Technology of China, P.R. China.*

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Abstract. We present generalized and unified families of $(2n)$-point and $(2n-1)$-point p-ary interpolating subdivision schemes originated from Lagrange polynomial for any integers $n \geq 2$ and $p \geq 3$. Almost all existing even-point and odd-point interpolating schemes of lower and higher arity belong to this family of schemes. We also present tensor product version of generalized and unified families of schemes. Moreover error bounds between limit curves and control polygons of schemes are also calculated. It has been observed that error bounds decrease when complexity of the scheme decrease and vice versa. Furthermore, error bounds decrease with the increase of arity of the schemes. We also observe that in general the continuity of interpolating scheme do not increase by increasing complexity and arity of the scheme.

AMS subject classifications: 65D17, 65D07, 65D05

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1. Introduction

Computer Aided Geometric Design (CAGD) is a branch of applied mathematics concerned with algorithms for the design of smooth curves and surfaces and for their competent mathematical demonstration. Subdivision schemes have become a very celebrated research area in CAGD and become a very famous modeling tool of curves and surfaces because of its potential to handle arbitrary topology. To save a smooth object which is created by means of subdivision, one only requires storing a coarse approximation and the subdivision scheme constructing the object. This reality makes subdivision a useful tool in computer aided geometric design. In fact a subdivision

*Corresponding author. *Email addresses:* ghulam.mustafa@iub.edu.pk, mustafa_rakib@yahoo.com (G. Mustafa), pakeeza_ashraf@yahoo.com (P. Ashraf), dengjs@ustc.edu.cn (J. Deng)

scheme describes a curve from a primary arbitrary given control polygon by continuously subdividing them according to particular designed refining rules, such that the limiting curve can attain certain smoothness and continuity to meet the requirements of applications. In short one can develop complex smooth curves in a sensibly expected way from quite simple control polygons. Before giving the literature survey we first explain some basic terminologies:

- The number of points inserted at level $k+1$ between two consecutive points from level k is called *arity* of the scheme. If the number of points inserted are even then scheme is called *even-ary* scheme and if number of points inserted are odd then scheme is called *odd-ary* scheme.

- The number of points involved in the convex combination to insert a new point at next subdivision level is called *complexity* of the scheme. If the number of points involved are even then scheme is called *even-point* scheme otherwise it is called *odd-point* scheme.

The concept of subdivision has been first initiated by de Rham [17]. Later on, Deslauriers and Dubuc [2] presented b-ary 2N point schemes derived from polynomial interpolation. Dyn et al. [3] presented 4-point binary interpolating scheme with parameter. Brief review of higher arity schemes having even-point complexity is presented below. Ko et al. [11] introduced even point binary and ternary interpolating symmetric subdivision schemes. Mustafa and Khan [13] introduced a new 4-point C^3 quaternary approximating subdivision scheme. Lian [8] introduced 4-point and 6-point a-ary schemes. Lian [10] offered $2m$-point non-parametric interpolating even and odd-ary schemes for curve design. Zheng et al. [20] offered ternary even symmetric $2n$-point subdivision scheme. Zheng et al. [18] presented p-ary subdivision generalizing B-splines. Mustafa and Najma [14] unified all existing even-point interpolating and approximating schemes by offering general formula for the mask of (2b + 4)-point even-ary subdivision scheme.

Now we present brief review of higher arity schemes having odd-point complexity. Hassan and Dodgson [5] offered ternary and three-point univariate subdivision schemes. Hassan et al. [6] also presented 4-point ternary interpolating subdivision scheme. Lian [9] introduced 3-point and 5-point a-ary schemes. Lian [10] offered $(2m+1)$-point non-parametric interpolating odd-ary schemes for curve design. Zheng et al. [19] constructed $(2n-1)$-point ternary interpolatory subdivision schemes by using variation of constants. Aslam et al. [1] presented an explicit formula which unifies the mask of $(2n-1)$-point interpolating as well as approximating schemes. Mustafa et al. [16] presented an explicit formula for the mask of odd-points n-ary (for any odd $n \geq 3$) interpolating subdivision schemes. This formula unifies the schemes of [9, 10, 19] and many other schemes.

Zorin and Schröder [21] presented a unified framework for construction of an increasing sequence of alternating primal and dual quadrilateral subdivision schemes based on averaging approach. Starting with vertex split, they constructed variants