Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2014.1326nm

Vol. 7, No. 3, pp. 334-355
August 2014

A Nonlocal Total Variation Model for Image Decomposition: Illumination and Reflectance

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Received 27 August 2013; Accepted (in revised version) 7 February 2014

Abstract. In this paper, we study to use nonlocal bounded variation (NLBV) techniques to decompose an image intensity into the illumination and reflectance components. By considering spatial smoothness of the illumination component and nonlocal total variation (NLTV) of the reflectance component in the decomposition framework, an energy functional is constructed. We establish the theoretical results of the space of NLBV functions such as lower semicontinuity, approximation and compactness. These essential properties of NLBV functions are important tools to show the existence of solution of the proposed energy functional. Experimental results on both grey-level and color images are shown to illustrate the usefulness of the nonlocal total variation image decomposition model, and demonstrate the performance of the proposed method is better than the other testing methods.

AMS subject classifications: 68U10, 65K05, 65N21

Key words: Image decomposition, illumination, reflectance, nonlocal total variation, iterative method.

1. Introduction

In this paper, we address the problem of decomposing an image into its illumination and reflectance components. The illumination component is modeled as the amount of light intensity onto the objects in the image scene. The reflectance component is related to the light source reflected based on the nature of the objects in the image scene. There are many applications of such image decomposition like image enhancement and shadow removal, see [1–5].

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The recovery of the illumination and reflectance components from a given image is known to be an ill-posed problem [6, 7]. Land and McCann's Retinex algorithm [6, 8, 9] successfully decompose an image into the illumination and reflectance components. The Retinex algorithm is based on random walk and is a path-based algorithm. In [10–13], the researchers further proposed to develop recursive algorithms via recursive matrix operations to perform image decomposition into illumination and reflectance components. The efficiency of recursive algorithms is better than that of the path-based methods. Recently, several variational frameworks for Retinex theory are studied [1–5, 14–24]. In the setting, a mathematical optimization problem is formulated by decomposing an input image into the reflectance and illumination components, where their pointwise multiplication between these two components is assumed to be given input image. In [5], Kimmel et al. assumed that the illumination component was smooth and employed such smoothness condition in the objective function. In [20], bilateral filters were also used to construct regularization terms to perform image decomposition. In [3, 4, 15–19], the illumination component was assumed to be smooth and the reflectance component was assumed to be a piecewise constant function. According to these two assumption, a Poisson equation was set up for image decomposition. In [21], Ma and Osher proposed to use total variation and nonlocal total variation regularization models for Retinex theory. In their method, Bregman iteration was used to solve their models. In [22], Ma et al. further proposed a L_1-based variational model which is focused on recovering the reflectance component. In [23], Zosso presented a unifying framework for Retinex theory that is able to study many of the existing Retinex implementations including the methods in [21] and [22]. In [24], Ng and Wang developed and studied a total variation model for image decomposition which considers both reflectance and illumination components in the objective function.

The main aim of this paper is to use nonlocal bounded variation (NLBV) techniques to decompose an image intensity into the illumination and reflectance components. By considering spatial smoothness of the illumination component and nonlocal total variation (NLTV) of the reflectance component in the decomposition framework, an energy functional is constructed. The motivation behind is that the reflectance component contains a sparse set of texture patches in the space of nonlocal bounded variation functions. As an example, we compare in Fig. 1 the image decomposition of using the NLTV model and that of using the TV model [24]. We see from the figure that the reflectance component by the NTLV model (especially the upper region) is visually sharper and clearer than that by the TV model[†].

There are two main contributions in this paper. Firstly, we establish the theoretical results of the space of NLBV functions such as lower semicontinuity, approximation and compactness. These essential properties of NLBV functions are important tools to show the existence of solution of the proposed energy functional. To the best of our

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[†] Here we have used the best set of parameters chosen in [24] for the TV image decomposition model ($c_1 = 1$, $c_2 = 0.1$ and $c_3 = 10^{-5}$), and the best set of parameters for the NLTV model ($c_1 = 1$, $c_2 = 1$ and $c_3 = 10^{-5}$).