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A Robin-Robin Domain Decomposition Method for a Stokes-Darcy Structure Interaction with a Locally Modified Mesh

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Abstract. A new numerical method based on locally modified Cartesian meshes is proposed for solving a coupled system of a fluid flow and a porous media flow. The fluid flow is modeled by the Stokes equations while the porous media flow is modeled by Darcy's law. The method is based on a Robin-Robin domain decomposition method with a Cartesian mesh with local modifications near the interface. Some computational examples are presented and discussed.

AMS subject classifications: 65N30, 65N50, 65N55, 76D07, 76S05

Key words: Stokes-Darcy fluid structure interactions, Robin-Robin domain decomposition method, body fitted mesh, locally modified mesh, BJS interface condition.

1. Introduction

The coupled Stokes-Darcy system has recently attracted much attention among researchers. Its applications include flows across interfaces between soil and surface, in areas from oil extraction to bio-medicine. Although individually, the equations for the Stokes-Darcy flows are straightforward and well established, when these two PDE systems are coupled across an interface, there are challenges. The interface conditions between these two systems are the key part. Several conditions have been proposed [2, 8, 14]. In this paper, we consider the well accepted Beavers-Joseph-Saffman (BJS) [8,9,14] interface condition. The existence and uniqueness of weak solutions for the Stokes-Darcy system with BJS interface condition have been proven [10]. Consider

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the coupled Stokes-Darcy system on a bounded domain $\Omega_p \cup \Omega_f \in \mathbb{R}^d$. The motion of the fluid in Ω_f is modeled by the Stokes equations

$$-\nabla \cdot \mathbf{T}(\mathbf{u}_f, p_f) = \mathbf{f}, \tag{1.1}$$

$$\nabla \cdot \mathbf{u}_f = 0, \tag{1.2}$$

where $\mathbf{u}_f$ is the fluid velocity, p_f is the kinematic pressure, and $\mathbf{f}$ is the body force. $\mathbf{T}(\mathbf{u}_f, p_f) = 2\nu\mathbf{D}(\mathbf{u}_f) - p_f\mathbf{I}$ is the stress tensor and $\mathbf{D}(\mathbf{u}_f) = \frac{1}{2}(\nabla\mathbf{u}_f + \nabla^T\mathbf{u}_f)$ is the strain rate tensor. ∇ and $\nabla\cdot$ represent gradient operator and divergence operator respectively. The parameter $\nu > 0$ in the stress tensor is the kinematic viscosity of the fluid.

In the porous media region Ω_p, the fluid motion is modeled by Darcy's law

$$\mathbf{u}_p = -\mathbf{K}\nabla\phi_p, \tag{1.3}$$

$$\nabla \cdot \mathbf{u}_p = 0, \tag{1.4}$$

where $\mathbf{u}_p$ is the fluid velocity, $\mathbf{K}$ is the hydraulic conductivity tensor, and ϕ_p is the hydraulic head.

On $\Gamma = \Omega_f \cap \Omega_p$, let $\mathbf{n}_f$ denote the unit outward normal vector from Ω_f at Γ and $\mathbf{n}_p$ denote outward normal vector from Ω_p at Γ. $\boldsymbol{\tau}_j$ $(j = 1, \cdots, d-1)$ represents unit tangential vectors on Γ following right hand rule. See Fig. 1 as an example of the domain. Along the interface Γ, if we assume the nondimensional porosity of the Darcy region is 1, we have the mass conservation condition across Γ:

$$\mathbf{u}_f \cdot \mathbf{n}_f = -\mathbf{u}_p \cdot \mathbf{n}_p. \tag{1.5}$$

The second interface condition is the balance of normal stress across Γ:

$$-\boldsymbol{n}_f \cdot (\mathbf{T}(\mathbf{u}_f, p_f) \cdot \mathbf{n}_f) = g\phi_p, \tag{1.6}$$

where g is the acceleration parameter. As the fluid is viscous, a condition for tangential fluid velocity is needed [10]. A simple assumption is free slippage along Γ, $\boldsymbol{\tau}_j \cdot \mathbf{u}_f = 0$.

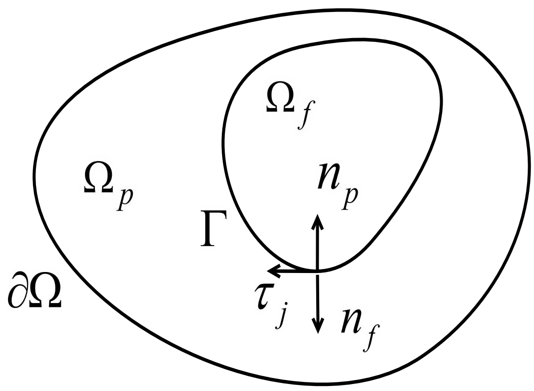

Figure 1: Sketch of a free flow region Ω_f, a porous media region Ω_p, and the interface Γ, the normal direction n_f and tangential direction τ_j, as well as boundary $\partial\Omega$.