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Unconditionally Stable Pressure-Correction Schemes for a Linear Fluid-Structure Interaction Problem

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Abstract. We consider in this paper numerical approximation of the linear Fluid-Structure Interaction (FSI). We construct a new class of pressure-correction schemes for the linear FSI problem with a fixed interface, and prove rigorously that they are unconditionally stable. These schemes are computationally very efficient, as they lead to, at each time step, a coupled linear elliptic system for the velocity and displacement in the whole region and a discrete Poisson equation in the fluid region.

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Key words: Fluid-structure interaction, pressure-correction, stability analysis.

1. Introduction

Fluid-Structure Interaction (FSI) plays an important role in many scientific/engineering applications, e.g., design of engineering systems, blood flow in human arteries, etc. It has been extensively studied in recent years both analytically and computationally (cf. [5, 7, 9, 14] and the references therein).

The fluid velocity, pressure and structure displacement in the FSI problems are coupled together, making it difficult to solve numerically. For fluid problems, an effective approach to decouple the computation of the pressure from that of the velocity is to use a so-called projection type method, originally proposed by Chorin and Temam in the late 60's. A comprehensive review on various projection type methods can be found in [12]. However, a main difficulty in the design of a projection method is what boundary condition to use for the pressure at the interface. It is well known that a proper boundary condition, at the Dirichlet part of the boundary, for the pressure Poisson equation in a projection type method is the homogeneous Neumann boundary condition. Indeed, most existing projection type schemes (cf., for instance, [1, 3, 10]) for

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FSI problem also use, explicitly or implicitly, Neumann type boundary condition for the pressure Poisson equation at the Dirichlet part of the boundary as well as at the interface. However, imposing a Neumann type boundary condition for the pressure at the interface appears to affect, in certain degree, the stability of the scheme, and we are not aware of any proof of unconditional stability for this type of projection scheme for the FSI problem, although a conditional stability has been proven in [10].

In [11], the authors proposed and analyzed pressure-correction projection schemes for Navier-Stokes equations with open boundary where the usual stress-free boundary condition is applied. It is shown that the proper boundary condition at the open boundary is of Dirichlet type instead of Neumann type. Two schemes are constructed in [11]. One is based on the standard pressure-correction which leads to poor accuracy at the open boundary, the other is based on the rotational pressure-correction and with a proper Dirichlet boundary condition at the open boundary. It is shown in [11] that both the standard and rotational pressure-correction projection schemes, when applied to the time-dependent Stokes problem, are unconditionally stable, but the rotational version leads to much better accuracy. Since one of matching interface condition for the FSI problem is related to the stress, it is sensible to extend the approach in [11] for problems with open boundary to the FSI problem.

In this paper, we shall construct a different class of projection semi-implicit schemes which decouple the computation of pressure from that of the velocity and structure displacement. Our schemes are computationally very efficient. More precisely, in the first step of our schemes, we solve a coupled, but elliptic, system for an intermediate fluid velocity and the structure displacement, then in the second step, we solve a Poisson equation for the fluid pressure and obtain the fluid velocity with a simple correction. Furthermore, we shall also prove rigorously that these schemes are unconditionally stable.

To illustrate the idea, we consider in this paper a simple model of the FSI problem where the movement of the interface is assumed infinitesimal so the interface is treated as fixed. This linear FSI problem captures many of the essential difficulties of the more general FSI problems with moving interface.

The rest of the paper is organized as follows. In the next section, we describe the governing equations for our FSI model, formulate its weak form and the energy dissipation law. In Section 3, we construct standard and rotational pressure-correction scheme for the FSI problem, and prove their unconditional stability. Then, in Section 4, we describe a Fourier-Legendre method for a special case when the domain is a periodic channel. We present some numerical results in Section 5 to validate our numerical schemes and to demonstrate their temporal accuracy. Some concluding remarks are given in Section 6.

2. Governing equations

We consider the following model for interaction of a viscous fluid with an elastic body in a two- or three-dimensional bounded domain Ω, with the fluid region Ω_f, the