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Local Fourier Analysis for Edge-Based Discretizations on Triangular Grids

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Abstract. In this paper, we present a local Fourier analysis framework for analyzing the different components within multigrid solvers for edge-based discretizations on triangular grids. The different stencils associated with edges of different orientation in a triangular mesh make this analysis special. The resulting tool is demonstrated for the vector Laplace problem discretized by mimetic finite difference schemes. Results from the local Fourier analysis, as well as experimentally obtained results, are presented to validate the proposed analysis.

AMS subject classifications: 65F10, 65N22, 65N55

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1. Introduction

In the numerical simulation of problems modeled by partial differential equations, the linear system solvers play a key role. Efficient algorithms enable large-scale computations with a satisfactory computing time and memory consumption. As is well-known, multigrid methods are among the most powerful techniques for this purpose. Since the 70's, when these methods [5,9,15,18] were developed, they have become very popular within the scientific community. Many approaches to multigrid theory have been investigated in the last years; among these, the technique of local Fourier analysis (LFA), introduced by Brandt [5,6], has become very successful, providing accurate predictions of performance for a variety of problems.

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LFA does not only provide accurate asymptotic convergence rates of the algorithms, but also is a useful technique for choosing suitable components for multigrid methods. Wienands and Joppich [19] provide a useful software tool for experimenting with Fourier analysis. Recent advances in this context include LFA for multigrid as a preconditioner [20], for triangular meshes [7, 14], optimal control problems [3], and discontinuous Galerkin discretizations [8]. In this paper, we present a framework for performing the local Fourier analysis for edge based discretizations on triangular grids. Although for simplicity in the presentation, we restrict ourselves to the two-dimensional case, we would like to emphasize that the technique presented here is easily extended to the three-dimensional case.

Mainly as a result of its good computational properties, edge-based discretizations have emerged widely in the simulation of many real applications, including electromagnetic field computations. Examples of such discretizations include Nédélec and Raviart-Thomas finite element methods and mimetic finite difference schemes. While in standard nodal discretizations, the unknowns are located at the nodes of a target grid, in case of edge-based schemes, the unknowns are associated with the edges of the corresponding mesh. Moreover, different stencils are associated with edges of different orientation. It is obvious that existing schemes for multigrid solution on nodal discretizations can not be used directly for edge-based schemes. Therefore, some efforts to implement multigrid solution schemes for edge based discretizations have been carried out recently, see for example [1, 2, 10, 11, 13]. For edge-based discretizations on triangular meshes, quantitative estimates of the multigrid methods as a function of the components chosen are missing in the literature. This question is at the focus of this paper.

Our aim is therefore to present a tool based on Fourier analysis, which does not only provide accurate asymptotic convergence rates, but also gives advice for an adequate composition of methods. To carry out this analysis in the framework of edge-based discretizations, several particular aspects have to be taken into account. First, the basis of LFA on triangular grids has to be considered, and also we have to deal with a discrete operator which is defined in a different way depending on the orientation of the edges, as we will see. To illustrate this analysis, mimetic finite difference schemes will be considered as example. This discretization is based on the discrete analogies of first-order differential operators, div, **grad**, rot and **curl**, that satisfy discrete analogies of the theorems of vector analysis, see for example [16]. Notice that other approaches e.g., the electromagnetic FIT [17] or Whitney element methods [4], have also been recognized as mimetic discretizations, see [12] for a review of all these techniques.

The remainder of the paper is organized as follows. In Section 2, a framework for the local Fourier analysis for general edge-based discretizations is developed. Section 3 introduces the mimetic finite difference scheme considered here to show the suitability of the presented analysis. To this purpose, in Section 4 some results of LFA for this type of discretization for a vector model problem, more concretely for the vector Laplace operator, are presented. Finally, in Section 5 some conclusions are drawn.