Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2015.m1334

Vol. **8**, No. 3, pp. 383-405
August 2015

A Multilevel Correction Method for Steklov Eigenvalue Problem by Nonconforming Finite Element Methods

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Received 18 November 2013; Accepted (in revised version) 28 April 2014

Abstract. In this paper, a multilevel correction scheme is proposed to solve the Steklov eigenvalue problem by nonconforming finite element methods. With this new scheme, the accuracy of eigenpair approximations can be improved after each correction step which only needs to solve a source problem on finer finite element space and an Steklov eigenvalue problem on the coarsest finite element space. This correction scheme can increase the overall efficiency of solving eigenvalue problems by the nonconforming finite element method. Furthermore, as same as the direct eigenvalue solving by nonconforming finite element methods, this multilevel correction method can also produce the lower-bound approximations of the eigenvalues.

AMS subject classifications: 65N30, 65N25, 65L15, 65B99

Key words: Steklov eigenvalue problem, multilevel correction, nonconforming finite element method.

1. Introduction

Steklov eigenvalue problem appears in a number of applications such as the antiplane shearing on a system of collinear faults under slip-dependent friction law [11], surface waves [5], the vibration modes of a structure in contact with an incompressible fluid [6], stability of mechanical oscillators immersed in a viscous fluid [15], vibrations of a pendulum [1], eigen oscillations of mechanical systems with boundary conditions containing the frequency [18].

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The two-grid method for solving eigenvalue problems has been proposed and analyzed by Xu and Zhou in [31]. The idea of the two-grid comes from [29, 30] for nonsymmetric or indefinite problems and nonlinear elliptic equations. Since then, there have existed many numerical methods for solving eigenvalue problems based on the idea of two-grid method (see, e.g., [2, 13, 19, 27, 32, 34, 35] and the references cited therein). The applications of the two-grid method to the Steklov eigenvalue problem have been investigated in [7, 8, 22].

In this paper, we present a multilevel correction scheme to solve the Steklov eigenvalue problem by nonconforming finite element methods. With the proposed method, the solution of the Steklov eigenvalue problem will not be much more difficult than the solution of the corresponding source problem. The correction method for eigenvalue problems in this paper is based on a series of finite element spaces with different levels of accuracies which are related to the multilevel method (c.f. [28]).

The standard Galerkin finite element method for eigenvalue problems has been extensively investigated, e.g. Babuška and Osborn [3, 4], Chatelin [12], Yang and Chen [33] and references cited therein. Here we adopt some basic results in these papers for our analysis. The corresponding error estimates of the proposed multilevel correction scheme by nonconforming finite element methods which is introduced here will be analyzed. Based on the analysis, the method can reduce the errors of the eigenpair approximations after each correction step. The multilevel correction procedure can be described as follows: (1) solve the eigenvalue problem in the initial nonconforming finite element space; (2) solve an additional source problem in an finer nonconforming finite element space using the previous obtained eigenvalue multiplying the corresponding eigenfunction as the load vector; (3) solve the eigenvalue problem again on the finite dimensional space which is constructed by combining a very coarse conforming finite element space with the obtained eigenfunction approximation in step (2). Then go to step (2) for the next loop.

In this method, instead of solving the Steklov eigenvalue problem in the finest nonconforming finite element space, we solve a series of boundary value problems in a series of nonconforming finite element spaces and a series of eigenvalue problems in a very coarse conforming linear finite element space plus one dimensional eigenfunction space. As known to all, there exists efficient preconditioner for solving boundary value problems. So this correction method can improve the overall efficiency of solving the Steklov eigenvalue problem by nonconforming finite element methods.

An outline of the paper goes as follows. In Section 2, we introduce the nonconforming finite element method for the Steklov eigenvalue problem and the corresponding error estimates. A type of one correction step is given in Section 3. In Section 4, we propose a type of multilevel correction algorithm to solve the Steklov eigenvalue problem by the nonconforming finite element methods. A lower-bound analysis of the eigenvalue approximations is given in Section 5. In Section 6, two numerical examples are presented to validate our theoretical analysis and some concluding remarks are given in the last section.