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A Lions Domain Decomposition Algorithm for Radiation Diffusion Equations on Non-matching Grids

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Abstract. We develop a Lions domain decomposition algorithm based on a cell functional minimization scheme on non-matching multi-block grids for nonlinear radiation diffusion equations, which are described by the coupled radiation diffusion equations of electron, ion and photon temperatures. The L^2 orthogonal projection is applied in the Robin transmission condition of non-matching surfaces. Numerical results show that the algorithm keeps the optimal accuracy on the whole computational domain, is robust enough on distorted meshes and curved surfaces, and the convergence rate does not depend on Robin coefficients. It is a practical and attractive algorithm in applying to the two-dimensional three-temperature energy equations of Z-pinch implosion simulation.

AMS subject classifications: 65M06; 65M55

Key words: Lions domain decomposition, radiation diffusion equations, non-matching grids, Schwarz algorithm.

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1. Introduction

Radiation transport in astrophysical phenomena and inertial confinement fusion is often modeled using a diffusion approximation. If the time scale is shorter than the equilibrium times of electron-ion and electron-photon, the radiation diffusion process needs three different temperatures in token of each element property. With three temperatures as independent variants, the equations contain heat diffusion and energy transfer between electrons, ions, and photons. They are highly nonlinear and form a strongly coupled system of parabolic differential equations. Many authors have studied the numerical solution of the radiation diffusion equations, they focus on the high

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order time integration methods [17, 19] and the nonlinear iteration solution methods on rectangular meshes [10, 13, 18].

In many application fields, the radiation diffusion problem often couples with the hydrodynamics problem, and the Lagrangian method or arbitrary Lagrangian Eulerian method is often used to solve hydrodynamics problems. Due to the deformation of complicated fluid flow, computational meshes are distorted. Numerous efforts have been devoted to efficient discretization methods for diffusion problems on distorted meshes, such as the local support operator scheme (LSOM) [16] and its modern version namely the mimetic finite difference scheme (MFD) [4, 11, 14], the multi-point flux approximation scheme (MPFA) [1,6,12], the nine-point scheme [23,25], the nonlinear monotone scheme [24, 29], and the cell functional minimization scheme (CFM) [26, 27].

In the numerical simulations of the practical physical problems, complex geometric computational domain and material surface sliding may lead to non-matching grids. In the framework of finite volume method, several discretization methods for non-matching grids have been developed [2, 3, 7], these methods deal with special non-matching grids with hanging nodes or numerical errors depend on the Robin parameter. In order to keep the original finite volume accuracy, a linear rebuilding along the interface is introduced [22]. In [28], we only use the L^2 projection operator on non-matching multiblock grids based on a CFM scheme in each block to get excellent results for a classical linear parabolic equation. Not only the new non-matching DDM algorithm keeps the accuracy of CFM scheme, but also numerical errors are independent of the Robin parameter. Theory analysis and numerical tests are presented.

Our objective is to develop the DDM numerical algorithm presented in [28] for radiation diffusion equations with three different temperatures to handle efficiently non-matching grids. Here, we focus on the numerical errors of the whole computational domain. Do not consider the convergence of the Schwarz method for non-matching grids. Moreover, usually the additive Schwarz method is replaced by much more efficient Krylov type methods and in addition, the small enough problems can also be solved by a direct method. Our main idea is to combine the Lions domain decomposition algorithm with the cell functional minimization (CFM) scheme, while satisfying the following properties. First, as sliding blocks are considered, the discretization in one block should not depend on the grid of the adjacent block. Second, there is a simple procedure to be used in the transmission condition of non-matching surfaces. Third, the Lions domain decomposition algorithm do not lose the finite volume accuracy on the whole computational domain. Finally, the algorithm should be robust enough on the distorted meshes and curved surfaces.

The rest of the paper is organized as follows. In the next section, the simplified three temperatures diffusion equations are described. In Section 3 the Lions domain decomposition algorithm are presented. In Section 4 the CFM scheme inside a subdomain is derived. In Section 5 how to construct discrete interface data on a general