Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2015.m1416

Vol. **9**, No. 1, pp. 87-110
February 2016

Positivity-Preserving Runge-Kutta Discontinuous Galerkin Method on Adaptive Cartesian Grid for Strong Moving Shock

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Received 01 June 2014; Accepted (in revised version) 16 January 2015

Abstract. In order to suppress the failure of preserving positivity of density or pressure, a positivity-preserving limiter technique coupled with h-adaptive Runge-Kutta discontinuous Galerkin (RKDG) method is developed in this paper. Such a method is implemented to simulate flows with the large Mach number, strong shock/obstacle interactions and shock diffractions. The Cartesian grid with ghost cell immersed boundary method for arbitrarily complex geometries is also presented. This approach directly uses the cell solution polynomial of DG finite element space as the interpolation formula. The method is validated by the well documented test examples involving unsteady compressible flows through complex bodies over a large Mach numbers. The numerical results demonstrate the robustness and the versatility of the proposed approach.

AMS subject classifications: 65M50; 65M60; 76L05

Key words: Discontinuous Galerkin method, adaptive Cartesian grid, positivity-preserving, immersed boundary method, complex geometry.

1. Introduction

Recently, the Cartesian grid methods have become very popular in computational fluid dynamics (see [1–11] and their references), because such methods do not suffer from the complex grid generation and grid management requirements which are inherent in other methods, and also these methods are easily extended to high order

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numerical schemes. Conceptually, the Cartesian grid approach is much simpler to be implemented than other grid methods. In general, solid bodies with a Cartesian grid for the partition of the flow field are cut out of a single static background mesh and their boundaries are represented by different types of cut cells. When cut cells become very small, however, degenerate cells will be encountered. In this situation, numerical instability may occur when an explicit time step scheme is used in numerical calculations. Some techniques have already been employed to overcome these problems along with time step stability restrictions [1, 6–9]. Although there are many different techniques, ghost cell method or immersed boundary method for its simplification still obtains many researcher's favourite [2–5, 10, 11]. In our recent article [12], we developed an adaptive Cartesian grid RKDG method combined with the ghost cell immersed boundary technique to deal with a complex geometry. This methodology was based on the image point ghost cell method [4] and used an inverse distance weighting interpolation formula to obtain the value at the image point. In this paper, we extend this idea and develop a new approach for immerse boundary treatment, in which the interpolation formula for cell solution polynomials is created on discontinuous Galerkin finite element space.

In practice, it is quite often to encounter the situation in which the density or pressure of the numerical solutions becomes negative [13–15]. For instance, highly energetic flows may contain regions with a dominant kinetic energy, and a relatively small internal energy which is easy to become negative in the simulation [16]. Another well-known example is the computational simulation of shock wave or gas detonation propagation through different geometries [15]. The shock diffraction may result in very low density and pressure [13–16]. In general, the most commonly used high order numerical schemes for solving Euler equations do not satisfy the positivity property, which may produce negative density or pressure and cause blow-ups of the numerical algorithm. The ad hoc methods in numerical strategy, which modify the computed negative density and/or the computed negative pressure to be positive, destroy not only a local and global conservation, but also often cause numerical instability [17]. Recently, based on certain Gauss-Lobatto quadratures and positivity-preserving flux, Zhang and Shu [13–15] used Lax-Friedrichs flux and successfully developed a positivity-preserving approach for high-order discontinuous Galerkin methods. Such an approach is also applied to unstructured meshes and p-adaptive numerical solutions by Kontzialis and Ekaterinaris [18]. In a recent paper of Wang *et al.* [16], they simplified the method and extended it to solve gaseous detonations. The aim of the present work, then, is to develop a simple approach under the adaptive Cartesian grid to simulate large Mach number flows with strong shock/obstacle interactions and shock diffraction. The present paper can be considered as a companion work to [12] on the so-called adaptive Cartesian grid RKDG methods for arbitrarily complex geometries. More specifically, in this paper we employed a simplified version of high-order positivity-preserving technique with h-adaptive RKDG method, and a modified version of the well-known Harten-Lax-van Leer contact numerical flux named as HLLC-HLL flux in [19] to remedy the numerical shock instability.