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Nonconforming Mixed Finite Element Method for Time-dependent Maxwell's Equations with ABC

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Abstract. In this paper, a nonconforming mixed finite element method (FEM) is presented to approximate time-dependent Maxwell's equations in a three-dimensional bounded domain with absorbing boundary conditions (ABC). By employing traditional variational formula, instead of adding penalty terms, we show that the discrete scheme is robust. Meanwhile, with the help of the element's typical properties and derivative transfer skills, the convergence analysis and error estimates for semi-discrete and backward Euler fully-discrete schemes are given, respectively. Numerical tests show the validity of the proposed method.

AMS subject classifications: 65N30, 65N15, 35J25

Key words: Maxwell's equations, Absorbing boundary condition, Nonconforming mixed FEM, Semi-discrete and fully discrete schemes.

1. Introduction

The Maxwells equations are the fundamental equations for understanding most electromagnetic and optical phenomena. The study of theoretical and numerical solutions of Maxwell's systems has been an attractive issue in computational mathematics [2, 7].

As we know, the numerical solutions of C^0-conforming vector nodal FEMs have an interesting history [11, 12] used to Maxwell's equations. Unfortunately, any C^0-conforming FEM will fail if the solutions do not belong to $H^1(\Omega)$, especially when the domain Ω is non-convex. This kind of wrong behavior has been overcome by H. Y. Duan in [13]. The critical idea is that the local L^2 projected program is utilized and two so-called mesh-dependent stabilization terms are added to the bilinear form and right-hand side respectively. It is inevitable that the computing cost is increased and the error estimations are very complicated during the process. Therefore, $H(curl)-$conforming edge element method is meeting and becoming popular.

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In the last thirty years, $H(curl)-$conforming edge elements have been invented to solve time-dependent Maxwell's equations, see e.g., [1–3, 7, 9, 18]. Monk [2] studied the equations by using Nédélec's conforming mixed FEMs approaching of time-dependet Maxwell's systems with perfect electric conductor (PEC) conditions, and derived the interpolation error estimation and Céa lemma, as well as the error estimations in $H(curl)$ norm. Furthermore, some wonderful results were obtained by Li et al. [17–19], in which both semi-discrete and fully-discrete FE schemes were constructed, and optimal order error estimates in the broken energy norm were proved. In [1], the interior penalty discontinuous Galerkin (DG) methods for the time-dependent Maxwell's equations in cold plasma were also developed. Some convergence analysis of an adaptive edge element method for Maxwell's equations have been explored in [21, 22] and references therein.

Since 1980's, nonconforming FEMs have been widely used in numerical solutions of partial differential equations. The first constructive theoretical and numerical analysis for Maxwell's equations can be found in [14–16], in which the Crouzeix-Raviart type nonconforming FE approximating to two dimensional $curl$-$curl$ and $grad$-div system was studied. And numerical experiments indicated that the traditional weak formula could not lead to a convergence scheme even if the mesh is refined. Therefore, the traditional weak formula was modified in [14–16] by adding penalty terms, which involves the tangent and normal jumps. The error estimates and numerical experiments showed that the modified form works well. The so-called crucial difference is that the piecewise $H(curl) \cap H(div)$ semi-norm, unlike the piecewise H^1 semi-norm, is too weak to control the jumps even if the mesh is refined. Hence the two terms involving the jumps must be included in the discretization so as to control the consistency error.

Through analyzing the error estimate of Maxwell's equations by Crouzeix-Raviart type nonconforming FEs, Shi et al. presented some work [23–25] where rectangular nonconforming FEs were constructed. In [23], the new nonconforming FEMs were proposed for approximating the Maxwell's equations on anisotropic meshes by mixed FE formulations in 2D, and 3D in [24]. In [25], the convergence analysis of Quasi-Wilson nonconforming FE approximation of Maxwell's equations was discussed under arbitrary quasi-uniform quadrilateral meshes. The subsequent work can also be found in [26], where a new mixed nonconforming FE space was constructed and applied to discuss the time-harmonic Maxwell's equations, and the analysis of extrapolation and superconvergence were also studied, respectively. Such these constructions have a same typical advantage, that is, the FEs' consistency errors are one order higher than their interpolation errors. Based on this crucial property, the traditional variational formula can be used without adding any 'stability' or 'penalty' term. However, the above researches are only carried out for PEC.

In practical computations, the electromagnetic fields are often truncated in a bounded domain with ABC, such as electromagnetic scattering problems. With the ABC, locally in space and time, there will arise two problems need to be considered: the well-posedness of the original problems in an artificial domain and reflection of waves at