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Spline Surfaces over Arbitrary Topological Meshes: Theoretical Analysis and Application

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Abstract. Based on polyhedral splines, some multivariate splines of different orders with given supports over arbitrary topological meshes are developed. Schemes for choosing suitable families of multivariate splines based on pre-given meshes are discussed. Those multivariate splines with inner knots and boundary knots from the related meshes are used to generate rational spline shapes with related control points. Steps for up to C^2-surfaces over the meshes are designed. The relationship among the meshes and their knots, the splines and control points is analyzed. To avoid any unexpected discontinuities and get higher smoothness, a heart-repairing technique to adjust inner knots in the multivariate splines is designed.
With the theory above, bivariate C^1-quadratic splines over rectangular meshes are developed. Those bivariate splines are used to generate rational C^1-quadratic surfaces over the meshes with related control points and weights. The properties of the surfaces are analyzed. The boundary curves and the corner points and tangent planes, and smooth connecting conditions of different patches are presented. The C^1-continuous connection schemes between two patches of the surfaces are presented.

AMS subject classifications: 65D18, 68U05

Key words: Polyhedral spline, inner knot, multivariate spline, control point, spline shape, C^k-surface, arbitrary topological mesh , rational bivariate quadratic surface, rectangular mesh.

1. Introduction

Surfaces with arbitrary topology play an important role in computer aided geometric design. Subdivision surfaces, as the main surfaces with arbitrary topology, are usually continuous and can be C^1-continuous as limiting results, with skillful subdivision methods. As limiting results, those subdivision surfaces in game design are superior to those in other designs where precision coordinates are necessary.

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Spline surfaces are another kind of surfaces. In 1946, Schoenberg set up the theory of univariate spline. With this theory, B-spline has been used widespreadly in [1-4], and NURBS curves became some of the most popular curves in computer aided geometric design. These successes stimulated various ideas and methods that surfaces could be designed similarly.

As tensor surfaces, NURBS surfaces are in form of tensor product of two univariate splines along two coordinates. And C^1-NURBS surfaces are rational piecewise polynomials of degree 4. Triangular Bézier surfaces are designed by generalizing Bézier curves with barycentric coordinates in the plane. Bézier curves is only a certain form of B-spline curves, however.

Naturally, bivariate splines can be directly used to design surfaces. Based on different ideas and rules, there are plenty of splines, such as simplex splines, box splines, half box splines and cone splines, or in general, polyhedral splines which are presented in [5] by Goodman (See [5] and its references for more details). With those splines over specific meshes, there are some related spline surfaces, such as box spline surfaces, *etc*.

For surfaces related to splines over arbitrary topological meshes, methods with specific techniques seem necessary. Starting with bivariate 3-directional quartic box spline, [6] proposed an algorithm for C^1-surfaces over arbitrary triangular control meshes. [7] introduced an algorithm for creating smooth spline surfaces over control triangular meshes capable of outlining arbitrary free-form surfaces with or without boundary. The resulting surface had a degree 4 parametric polynomial representation and was represented as a network of tangent plane continuous triangular Bézier patches. In [8], based on the generalized B-spline blending functions and G^1-continuous conditions, the blending matrices were derived. By means of the blending matrices, the control vertices in an irregular control polyhedron were converted to control points of piecewise B-spline patches.

A new scheme for constructing splines and spline surfaces with arbitrary topology was discussed. The second section reviewed the definition and basic properties of polyhedral splines in [5] and developed a definition of multivariate splines with given supports as well as concepts of boundary knots, inner knots and rational spline shapes, and analyzed the relationships among meshes, splines, knots, and control points. In the third section, C^0- and C^1-bivariate spline hybrid mesh surfaces were reviewed with degrees of one and two, respectively. To avoid some unexpected discontinuities and to get higher smoothness, a heart-repairing technique to adjust inner knots in the multivariate splines was designed in the fourth section. The fifth section was devoted to bivariate cubic splines and C^2-surfaces over arbitrary triangular meshes.

Based on the theory in sections from 2 to 5, bivariate quadratic C^1 splines over rectangular meshes were computed in [12]. With those splines, rational quadratic C^1-surfaces over rectangular meshes was developed.

In the sixth section, rational bivariate quadratic C^1-spline surfaces over rectangular meshes was presented. These surfaces are composed of patches. The formulas of the patches with local coordinates, related control points and weights were devel-

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