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Two-Stage Image Segmentation Scheme Based on Inexact Alternating Direction Method

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Abstract. Image segmentation is a fundamental problem in both image processing and computer vision with numerous applications. In this paper, we propose a two-stage image segmentation scheme based on inexact alternating direction method. Specifically, we first solve the convex variant of the Mumford-Shah model to get the smooth solution, the segmentation are then obtained by apply the K-means clustering method to the solution. Some numerical comparisons are arranged to show the effectiveness of our proposed schemes by segmenting many kinds of images such as artificial images, natural images, and brain MRI images.

AMS subject classifications: 65J22, 68U10, 90C25

Key words: Mumford-Shah model, Total variation, Image segmentation, K-means, Inexact alternating direction method.

1. Introduction

Image segmentation plays an important role in various applications, e.g., medical imaging and target identification [9, 19, 37], which is to part an image into different regions that correspond to distinct objects in the depicted scene. Approaches of segmentation models based on the calculus of variations and partial differential equations (PDEs) have been extensively studied in the past twenty years [7, 26, 30–32] due to their natural physical properties and flexible numerical methods. Among the best known and most influential examples is the Mumford-Shah (MS) model [26], which approximates an image by a piecewise smooth function with regular boundaries. This model can be briefly described as follows: Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with

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Lipschitz boundary, and $f(x):\Omega\rightarrow\mathbb{R}$ represents a given grayscale image. To find the segmentation of $f(x)$, Mumford and Shah in [26] proposed to minimize

$$E_{MS}(g,\Gamma)=\frac{\bar{\mu}}{2}\|g-f\|_{L^2}^2+\frac{\bar{\lambda}}{2}\int_{\Omega\setminus\Gamma}|\nabla g|^2dx+\mathrm{Length}(\Gamma), \tag{1.1}$$

where $\bar{\mu}$ and $\bar{\lambda}$ are positive weighted parameters used to control the contribution of each term, Γ is a compact curve in Ω, and $g:\Omega\rightarrow\mathbb{R}$ is continuous in $\Omega\setminus\Gamma$ but may be discontinuous across Γ.

The MS model (1.1) brings up a tough optimization problem. Over the years, numerous numerical algorithms have been proposed to solve it [1,4,14,25]. For example, the length term $\mathrm{Length}(\Gamma)$ was replaced by a phase field energy and it was approximated by a sequence of simpler elliptic variational problems [1]. Later, by using a family of continuous functions, some nonlocal approximation methods were proposed in [4,14,25]. Meanwhile, many people also tried to simplify the MS model (1.1). A useful simplification is to restrict the minimization results to take a finite number of values. The resulting model is commonly referred to as the piecewise constant MS model. The special case was studied by Chan and Vese [7] by restricting the solution to be two constants as the background and foreground. For more works on the general piecewise constant MS model, see [21,30–32], etc. However, the main drawback of these methods is usually getting stuck in the local minimization due to the nonconvexity of the MS model (1.1). So many methods such as graph cut methods [2,16] and convex relaxation methods [6] were proposed to overcome this drawback.

In this paper, instead of directly solving the MS model (1.1), we propose a two-stage image segmentation scheme. Specifically, the first stage considers to solve the convexification MS model (1.1) as

$$\min_g\left\{\frac{\bar{\mu}}{2}\|Hg-f\|_{L^2}^2+\frac{\bar{\lambda}}{2}\|\nabla g\|_{L^2}^2+\int_\Omega|\nabla g|dx\right\}, \tag{1.2}$$

where H can be the identity operator, the blurring operator or the generic linear operator. Different to the original MS model (1.1), here we omit the part Γ of the integral region in the second term due to the fact that the Lebesgue measure of the boundary curve Γ is zero. We also introduce a linear operator H to extend it to more general image segmentation problem in the first term. Once the smooth solution g in (1.2) is found, the second stage will use the K-means clustering method to obtain the segmentation image based on clustering the smooth solution g into different phases with suitable threshold(s).

The original idea of the two-stage image segmentation strategy as an extremely powerful method was proposed in [11,22,23]. However, their methods are only determined when the segmentation number K is given. Furthermore, the smoothing and thresholding are done alternatively after a number of iterations, which is computationally expensive. So it is very reasonable to independently obtain the smooth image g and thresholds. Hence our two-stage scheme is quite suitable for users to reveal different features within the image by choosing different thresholds.