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Finite Volume Element Methods for Two-Dimensional Three-Temperature Radiation Diffusion Equations

Yanni Gao[1], Xiukun Zhao[2], and Yonghai Li[1,*]

[1] *School of Mathematics, Jilin University, Changchun 130012, Jilin, People's Republic of China.*
[2] *Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600MB Eindhoven, The Netherlands.*

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Abstract. Two-dimensional three-temperature (2-D 3-T) radiation diffusion equations are widely used to approximately describe the evolution of radiation energy within a multi-material system and explain the exchange of energy among electrons, ions and photons. Their highly nonlinear, strong discontinuous and tightly coupled phenomena always make the numerical solution of such equations extremely challenging. In this paper, we construct two finite volume element schemes both satisfying the discrete conservation property. One of them can well preserve the positivity of analytical solutions, while the other one does not satisfy this property. To fix this defect, two as repair techniques are designed. In addition, as the numerical simulation of 2-D 3-T equations is very time consuming, we also devise a mesh adaptation algorithm to reduce the cost. Numerical results show that these new methods are practical and efficient in solving this kind of problems.

AMS subject classifications: 65N08, 65N12, 65N15

Key words: Two-dimensional three-temperature radiation diffusion equations; Finite volume element method; Energy conservation property; Repair technique; Mesh adaptation, Cutoff method.

1. Introduction

Radiation diffusion equations are widely used to simulate problems in inertial confinement fusion experiments, magnetic confinement fusion experiments, astrophysics problems, and so on. If radiation and material do not reach equilibrium, the equations are nonequilibrium radiation diffusion problems. 2-D 3-T radiation diffusion equations [12, 14–16, 25, 26] coupled with electron, ion and photon temperatures belong to nonequilibrium radiation diffusion problems. They are widely used to approximately describe the process of energy across multiple materials and the exchange of

*Corresponding author. *Email addresses:* `yonghai@jlu.edu.cn` (Y.-H. Li), `gaoyn10@163.com` (Y.-N. Gao), `ying509501@126.com` (X.-K. Zhao)

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energy among electrons, ions and photons. Because they have the characteristics of high nonlinearity, strong discontinuity and tight coupling, the numerical simulation of such equations is very challenging and interesting, which has attracted many authors' attention.

There are two key requirements desired to be satisfied in the numerical simulation of such problems. One is energy conservation property, the other is monotonicity, i.e. preserving positivity of analytical solutions. It is well known that the analytical solutions of the governing equations can satisfy some conservation forms inherently, but the numerical solutions can just satisfy the discrete analogue of these conservation forms [11, 13, 30]. So usually a discrete scheme is conservative, it means that this scheme can be expressed as the conservation form in a discrete sense. However, the numerical solutions of a conservative scheme are still very likely to appear negative solutions, which are not true because of the physics of the problem and will lead to the breakdown of computations. In order to deal with this problem, it is better to construct a discrete scheme which is unconditional monotone or monotone under less restrictive conditions [17, 18, 29, 32, 34, 39]. Monotone schemes are tightly connected to algebraic properties of the matrix of the discrete operator, and need impose severe restrictions on both meshes and problem coefficients. Therefore, the construction of monotone schemes is usually very difficult. Besides, some post-processing techniques such as the repair paradigms proposed in [19, 23, 31, 36] and the cutoff method analyzed in [24] can also be adapted to repair the negative solutions. The work mentioned above only focus on elliptic equations, diffusion problems with simple diffusion tensors and nonequilibrium radiation diffusion equations coupled with electron and ion temperatures. Next, we will introduce the existing work for 2-D 3-T radiation diffusion equations.

As to 2-D 3-T radiation diffusion equations, the relevant advances about numerical solutions can refer to [15, 26–28, 38]. In the framework of UG, the paper [26] successfully solved the problems by using the fully implicit finite volume scheme and parallel adaptive multigrid method on the triangular grids. The author discussed the influence of the particular time step chosen to computational efficiency detailedly. In the context of the symmetric finite volume scheme established in [27], two kinds of linearization methods, namely, Newton and "fixed-coefficient", have been researched in [28]. [15] constructed a symmetric finite volume element scheme which made the corresponding preconditioning technique applicable for 2-D 3-T radiation diffusion equations, and proposed a mesh adaptation approach based on monitor matrix. By this mesh adaptation, the energy conservation error and computation efficiency can be improved obviously. Through decoupling one temperature variable from the other two ones by a special coarsening strategy, [38] designed a two-level iterative method for solving the linear system discretized from 2-D 3-T radiation diffusion equations.

Finite volume element (FVE) methods are also called the generalized difference methods in China, belonging to the box schemes, or the control volume methods. Similar to the standard finite volume methods, we need to primarily integrate the governing equation on the control volume. Then, by using a finite element space to approximate

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