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Bivariate Polynomial Interpolation over Nonrectangular Meshes

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Abstract. In this paper, by means of a new recursive algorithm of non-tensor-product-typed divided differences, bivariate polynomial interpolation schemes are constructed over nonrectangular meshes firstly, which is converted into the study of scattered data interpolation. And the schemes are different as the number of scattered data is odd and even, respectively. Secondly, the corresponding error estimation is worked out, and an equivalence is obtained between high-order non-tensor-product-typed divided differences and high-order partial derivatives in the case of odd and even interpolating nodes, respectively. Thirdly, several numerical examples illustrate the recursive algorithms valid for the non-tensor-product-typed interpolating polynomials, and disclose that these polynomials change as the order of the interpolating nodes, although the node collection is invariant. Finally, from the aspect of computational complexity, the operation count with the bivariate polynomials presented is smaller than that with radial basis functions.

AMS subject classifications: 65D05

Key words: Bivariate polynomial interpolation; Nonrectangular meshes; Scattered data interpolation; The conformality of smoothing cofactor method; Radial basis function.

1. Introduction

The multivariate polynomial interpolation in the form of non-tensor product has many special features which do not arise in the simple univariate case directly. In fact, the geometry of the set of interpolation points (called nodes) is crucial for determining the solvability of the interpolation problem. Thus the challenging subject on multivariate interpolation has attracted much attention in the past. Salzer [14] pointed out that

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the existing interpolation formulae for more than one dimension possessed the desirability of at least three properties, i.e, truly irregular distribution of nodes, fairly simple recursion scheme for calculating successive coefficients, and limiting confluent forms. And he proposed a new bivariate divided difference algorithm to construct the bivariate polynomial interpolation over ortho-triples, the nodes of which were arranged in groups of three to form L-like configurations.

During many years, a vast amount work of multivariate polynomial interpolation has been done. Mazroui etc. [10] studied a recursive method for the construction of a Hermite spline interpolation. Madych [9] obtained an estimate of the $L^p(\Omega)$ norm of u in terms of the l^p norm of the values and its high-order derivatives, which is useful in obtaining error estimates for certain interpolation schemes. Dyn and Floater [5] studied multivariate polynomial interpolating on lower sets of points, which were expressed as the union of blocks of points. And Bailey [2] gave an equivalence between existence of particular exponential Riesz bases and existence of certain polynomial interpolants. Cui and Lei [4] solved one of the key problems in multivariate Birkhoff interpolation, which is to determining a monomial basis spanning the interpolation space. Allasia and Bracco [1] made a further study for Hermite-Birkhoff interpolation to multivariate real functions on scattered data by constructing a class of cardinal basis functions. And Chai etc. [3] proposed two algorithms to compute the basis of the minimal interpolation space and the lower interpolation space respectively.

However, to be mentioned, classical multivariate interpolation has always been restricted on the study of the case of tensor product [16], while multivariate interpolation in the form of non-tensor product has received not deep but constant attention [6] except scattered data interpolation with the well-known radial basis functions [21]. In the case of non-tensor-product-typed multivariate interpolation, on one hand, classic Bezout Theorem has played a crucial pole [16]. On the other hand, much attention has been paid on the study of convex preserving scattered data interpolation, which is due to multivariate splines [20]. Lai [7] used bivariate C^1 cubic splines to deal with convexity preserving scattered data interpolation problem. And to improve the order of approximation, Zhou and Lai [22] proposed several new schemes. Zhu and Wang [23] presented a method to construct Lagrange interpolation sets for bivariate spline spaces on cross-cut partitions by using interpolation along a piecewise algebraic curve. The above construction of the bivariate polynomial interpolation schemes needs the spline bases in the minimal supports, which can be worked out by means of the Conformality of Smoothing Cofactor Method. For the computation of the spline bases, one can see [8, 11–13, 17, 20].

Being Inspired by the construction of bivariate polynomial interpolation scheme over ortho-triples in [14], we have considered the case of bivariate continued fraction interpolation over the ortho-triples [18, 19], which is based on a new partial inverse divided differences. However, to the best of our knowledge, there are few papers on the study of bivariate interpolation over general triples or nonrectangular mesh. Thus with inspiration of Salze's thoughts [14], we have continued the study of the subject and have obtained achievements, some of which are shown in this paper.