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A Multiple Interval Chebyshev-Gauss-Lobatto Collocation Method for Ordinary Differential Equations

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Abstract. We introduce a multiple interval Chebyshev-Gauss-Lobatto spectral collocation method for the initial value problems of the nonlinear ordinary differential equations (ODES). This method is easy to implement and possesses the high order accuracy. In addition, it is very stable and suitable for long time calculations. We also obtain the hp-version bound on the numerical error of the multiple interval collocation method under H^1-norm. Numerical experiments confirm the theoretical expectations.

AMS subject classifications: 65L05, 65L60, 41A10, 65L70

Key words: Multiple interval Chebyshev-Gauss-Lobatto spectral collocation method, nonlinear ordinary differential equations, error analysis.

1. Introduction

Spectral methods possess high order accuracy, and have become increasingly popular in spatial discretization of partial differential equations (PDEs), see, e.g., [3–6, 8, 10, 21]. For time-dependent PDEs, one usually uses spectral methods to approximate the solutions in space and finite difference approaches to march the solutions in time. This tactic results in an unbalanced scheme: it has infinite accuracy in space and finite accuracy in time.

To overcome this disadvantage, some authors developed spectral methods for time-discretization of time-dependent PDEs, see, e.g., [1, 2, 7, 9, 17, 20, 22–25, 32, 33]. Moreover, some high order numerical methods for the initial value problems of ODEs are also established. For instance, Wihler [29] presented the continuous hp-Galerkin finite element time-stepping methods for the initial value problems of ODEs, Guo *et al.* [11–15, 26, 32, 33] designed several Legendre and Laguerre spectral collocation methods for the initial value problems of ODEs, and Yang and Wang [30] proposed a

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Chebyshev-Gauss spectral collocation method for the initial value problems of ODEs. Besides, Wang *et al.* [27,31,34] developed several Legendre spectral collocation methods for the nonlinear delay differential equations. The interested reader may also refer to Kanyamee and Zhang [18] and the references therein for other related works.

The aim of this paper is to propose a multiple interval Chebyshev-Gauss-Lobatto spectral collocation method for the initial value problems of ODEs:

$$\begin{cases} u'(t) = f(u(t),t), & t \in [0,T], \\ u(0) = u_0, \end{cases} \tag{1.1}$$

where f is a given function, and u_0 is the initial data. We approximate the solution by a finite Chebyshev series, and collocate the numerical scheme at Chebyshev-Gauss-Lobatto points. We also propose an efficient algorithm and present the error estimate for the hp-version of the multiple interval collocation method. Numerical results exhibit that the scheme is stable for long-time calculations and possesses high order accuracy. Moreover, it is also particularly attractive for ODEs with highly oscillating solutions, steep gradient solutions and nonsmooth solutions.

We highlight the main differences between our strategy and the existing ones as follows:

(i) We collocate the numerical scheme at Chebyshev-Gauss-Lobatto points, and fully analyze and characterize the hp-convergence of the multiple interval scheme under H^1-norm (the interplay between h and p can significantly enhance the numerical accuracy). The existing work [30] considered the Chebyshev-Gauss spectral collocation method, and only analyzed the numerical error for the single step scheme under the weighted Sobolev space.

(ii) We use the Chebyshev expansions in each sub-step (known to be much stable than the usual Lagrange approach [21]), which lead to quite neat implementation through manipulating the expansion coefficients of the consecutive steps (see (2.36) below). The nodes and weights of Chebyshev-Gauss-Lobatto quadratures are given explicitly, avoiding the potential loss of accuracy (compared with Legendre and Laguerre quadratures). Particularly, the algorithm can be implemented efficiently by using fast Chebyshev transform.

This paper is organized as follows. In the next section, we propose the multiple interval Chebyshev-Gauss-Lobatto spectral collocation scheme, and present some approximation results on the Chebyshev-Gauss-Lobatto interpolation. The convergence analysis for the suggested method is given in Section 3. Numerical experiments are carried out in Section 4, which confirm the theoretical expectations. The final section is for some concluding remarks.

2. Multiple interval Chebyshev-Gauss-Lobatto collocation method

In this subsection, we propose a multiple interval Chebyshev-Gauss-Lobatto collocation method for (1.1).