Exploring Geometric Conjectures with the help of a Learning Environment - A Case Study with Pre-Service Teachers

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Abstract

Several researches show the potential of using new information and communication technologies (ICT) in the teaching and learning of mathematics. One of the branches in the teaching of mathematics that, over time, has deserved a special mention, is geometry, namely with the use of dynamic geometry systems (DGS). A learning environment for teaching and learning geometry, associated with tasks of an exploratory and investigative nature, tends to favor the discovery of properties and geometric relationships, which benefits the acquisition of knowledge and the production of evidence. In this article the learning environment Web Geometry Laboratory (WGL) was used in a case study for the exploration of visual proofs. The study was done with pre-service teachers. Conclusions are drawn and future work is foreseen.

1 Introduction

Mathematical proof remains a means to everyone to prove to someone the truthfulness of an out tasks [5, 9, 17]. On the one hand we have a formal axiomatic theory, with a clear set of axioms (set of truths about a given reality, organized into concepts based on primitive terms) and lemmas, where geometric conjectures can be formally proved, on the other hand we have geometric models, geometric constructions with its appealing, intuitive, visual rendering. Dynamic geometry offers the ability to experiment, to use strategies, guess, argue and deduce mathematical properties. The concrete manipulation of objects enables abstract manipulation, thus making more accurate deductions that lead to the development of mathematical reasoning [17]. The use of geometric construction to infer a given geometric property can be misleading, the construction is only a particular case of the geometric property that is being considered. The use of an e-learning environment incorporating a dynamic geometry system (DGS) for the learning of geometry is, in our opinion, a very important help, it will facilitate the learning process. The DGS allow to, dynamically, explore many instances (constructions) for a given conjecture, it is not a formal proof yet, but it encourages the elaboration of conjectures, helping the progress in mathematical communication, and developing mechanism of deductive reasoning.

With the advances in the distance education support tools and the Internet, it has become possible to disseminate knowledge extremely quickly and to meet the request of courses with flexible time and location. In this context, virtual environments become classrooms, where students and teachers communicate and interact through resources such as chats, emails and collaborative tools.

1.1 Collaborative Learning

Collaborative learning is defined as an approach to teaching and learning, involving groups of students working together, to develop a set of interpersonal skills, such as effective communication, negotiation, conflict resolution, decision making, leadership, personal responsibility and teamwork, exchanging knowledge and experiences, to solve a given problem, complete a task, or create a product [1, 16]. It is a possible strategy for different levels of education [29]. With the development of technologies, learning environments are enriched tools that allow sharing experiences and communications among its users. Collaborative learning computational systems become central to online education [24]. Collaborative work favours training in the capacity for synthesis, critical thinking, and the ability to summarize ideas or conjectures. The new technologies, in general, assume a preponderant role in other areas, with emphasis on the development of the problem-solving capacity due to the versatility that they show in the approach of different situations

1.2 Dynamic Geometry Tool

The role of information and communication technology (ICT) in the classroom had increased in the recent years [28]. The use of ICT tools represents a technological support for the visualisation of abstract concepts, allowing the production of mental models of the concept and assuming a more active role of the student in her/his learning process. With computer programs, students interact with educational material to develop the skills needed to solve problems using mathematics. In the area of geometry, the DGS are already well-known tools, steadily, but surely, substituting the ruler and compass tools. Many different DGS are currently available, e.g. *Cabri* [18], *C.a.R.* [10], *Cinderella* [23], *GeoGebra* [11] and *The Geometer's Sketchpad* [12][1].

One of the advantages of dynamic geometry programs is the accomplishment of tasks, not only exploring geometric situations, but also investigating situations that the tool itself fosters when moving objects, providing valuable support for students and teachers.

A case study for the exploration of visual manipulations and the formal validation counterpart is presented in this article. The *Web Geometry Laboratory (WGL)* [21], an e-learning collaborative and adaptive Web environment for geometry, incorporates a DGS, with a support of a database where each user can save geometric constructions produced using the DGS. The *WGL*, with the incorporated DGS, *GeoGebra 5* [11], is used to explore the many possible configurations for a given geometric construction and its properties, but also to introduce the formal validation of those properties using the new `Prove` and `ProveDetails` commands of *GeoGebra.*

1.3 Automated Theorem Proving Tool

Geometry Automatic Theorem Provers (GATP) are computer programs used to formally prove theorems in geometry, e.g. *GCLC* [13] and *JGEX* [30][2]. A GATP processes a series of conjectures, hypotheses and axioms, written in a formal language that allows to express in a precise and unambiguous way the problem to be solved, to generate a proof that describes how and why the conjectures follow from the axioms and hypotheses in a way that can be understood by humans. The GATP can be seen as computer programs that shows whether a sentence, a conjecture, is a logical consequence of a set of axioms and hypotheses. The proof describes a sequence of steps that validates the conjecture. Some pioneering work in the development of automatic prover of geometry theorems were made by Gelernter in the 60s of the 20th century [8], developing in a very active area of research [2, 3, 20]. Demonstrations are indispensable for the expansion of mathematical knowledge; the simple

[1] https://cabri.com/; http://car.rene-grothmann.de/; https://www.cinderella.de/; https://www.geogebra.org/; http://www.dynamicgeometry.com/

[2] http://poincare.matf.bg.ac.rs/~janicic/gclc/; https://sourceforge.net/projects/jgex/