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Liberal Arts Education and Applied Mathematics

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Abstract

A liberal arts education has for centuries been seen both as the pinnacle of education and the bedrock on which is founded good citizenship and exemplary leadership. In recent times, the liberal arts and the sciences have been seen as being in opposition, with educational institutions, governments, and students feeling forced to choose between them. But in our increasingly data-driven yet interconnected world, we believe that a new conception of the liberal arts is needed, one which blends the quantitative and analytical skills of a scientific education with the qualitative and communicative skills of a traditional liberal arts education. The ideal blend of these skills is to be found in training in applied mathematics.

Keywords: liberal arts, applied mathematics, quantitative skills

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Introduction

Every once in a while, we hear the claims that while liberal arts education is the best education we can offer tomorrow's leaders (Ono, 2019; Zakaria, 2015), it is in danger (Connor, 1998), an existential risk sometimes blamed on a recent focus on science, technology, engineering, and mathematics (STEM) education (Shalneva, 2020). In this article, we argue that a narrow view of the liberal arts sets up an unnecessary conflict with the sciences, and that it is time we go back to the original ideals of liberal arts education. Even though a liberal arts training is much broader than specific academic disciplines, we will argue that there is an existing discipline which is suited for achieving the objectives of liberal arts education – applied mathematics.

Before we make our case, it is worth briefly stating the original ideals of liberal arts education. As outlined in Joseph's book (2002), the purpose of the original liberal arts education was to provide students a general set of skills considered to be important for city states in ancient Greece and for many citizenries in the intervening centuries. It is important to note that these skills are not specific to a particular profession, but rather had the broad education of an informed citizenry and civic leaders in mind. Indeed, the belief was that citizens could only fully participate in a "government by the people" if the citizenry shared a set of intellectual, moral, and aesthetic skills. In other words, liberal arts education is not meant to provide training for craftmanship, or to produce technicians, but to produce citizens and leaders. Indeed, training in the seven liberal arts was explicitly distinguished from that in the "utilitarian" (or practical) arts, and that in the fine arts (Joseph, 2002).

It is also important to mention that these general skills for liberal arts education may evolve while the ideals shall remain the same. They were historically the "skills of freedom" required for citizens and freemen, and nowadays the skills of freedom required to work in the rapidly changing modern employment environment. In ancient Greece, these skills were divided into two parts. In the first part, the trivium (from which we derive the word trivial, in the same sense of being preliminary), were grammar, rhetoric, and dialectic. The three skills in the trivium were considered the fundamental skills, and together they bestow the ability to speak correctively and persuasively. In the second part, the quadrivium, were arithmetic (the theory of number), music (the application of the theory of number), geometry (the theory of space), and astronomy (the application of the theory of space), which were for intellectual development and which all have an inherently mathematical nature. The English version of the liberal arts as developed in the early modern period emphasized a living experience provided by the college system of Oxford and Cambridge universities, and a Socratic development of knowledge through an associated tutorial system. The college system was adopted by Harvard, Yale and other liberal arts universities in the US.

As pointed out by Zakaria (2015), the modern liberal arts education retains the aims of the trivium, namely the development of the ability to speak well (or form written arguments) but not those of the quadrivium (music and scientific literacy). Furthermore, because of this narrowed version of the liberal arts, they have come to be seen as almost synonymous with the humanities and social science majors in many liberal arts colleges and universities (Connor, 1998).

We believe that if we are to restore the original ideals of liberal arts education, we should enhance the second set of skills on quantitative, logical, scientific reasoning. For example, we should add computer literacy and basic statistical training into the curriculum as well as general science education beyond astronomy, including biology and other life science subjects. We note that the liberal arts education can be made disciplinary-specific over a wide range of subjects. For social science and humanity majors, a liberal arts education will require a curriculum consisting of basic training in mathematical, computational, as well as general scientific reasoning. For science majors, a liberal arts education will require a curriculum consisting of training in basic writing and other communication skills. Our vision is an inclusive one in which barriers between the so-called "two cultures" ("The Two Cultures" is the first part of an influential 1959 Rede Lecture by British scientist and novelist C. P. Snow which were published in book form as The Two Cultures and the Scientific Revolution the same year.) of the arts and the sciences are broken down, and no student can remain inside a bubble of ignorance of either culture. A truly liberal education requires not only the skills of the trivium but also those of the quadrivium, arts and science combined.

As one of the original seven "arts", music has always played an important role in our daily life. It is hard to imagine a world without music. Yet, it has been left out of the curriculum of most of universities including liberal arts colleges. Here we are not talking about taking a course "How to Enjoy Classical Music" as a free elective. We are asking whether we should encourage playing music instruments as a requirement, similar to the ones on physical education. As playing an instrument can teach so many to an individual, such as discipline, time management, teamwork (if in a group or a band), not to mention psychological and emotional benefits it brings as well, it should be an easy sell, right?

However, the focus of this article is not on how to develop a curriculum for general liberal arts education nor discipline-specific liberal arts education in general. Our objective here is to argue that there exists already a discipline specific liberal arts education. The subject is Applied Mathematics.

Not so fast, you might object. How can mathematics, viewed by many as a difficult subject, and mathematicians, often painted as nerds who lack emotional intelligence, be put into the same sentence of liberal arts education, let alone be a poster child for illustrating the ideals of it? These questions are precisely what we would like to answer below and in the rest of the article.

One of the main characteristics that distinguishes liberal arts education from a more narrowly focused program is its breadth, and the freedom liberal arts students have. This is also where applied mathematics differentiates itself from most of other scientific disciplines and identifies itself more closely with liberal arts. As a scientific discipline, applied mathematics is a perfect example of a balance between breadth and depth.

But a liberal arts education is not just about the balance between breadth and depth. It is about what we hope to achieve, which is the ability to learn instead of specific skills. Since modern science including mathematics has developed into the stage that is beyond the understanding of the public, it has lost its original place in the ancient Greek liberal arts curriculum. Some eminent scholars such as Lawrence Summers (former Harvard president) and Shirley Tilghman (former Princeton president) have called for the return of science to liberal arts education (Zakaria, 2015). However, it must be done in the way that students learned the essence of scientific thinking, not by simply adding a few courses. A carefully designed "modern" applied mathematics program can achieve that, much like the current liberal arts education teaches students how to write, speak, and ultimately how to learn new materials that are not taught in any specific courses.

"A new era is at hand, an era that will call for broad knowledge and deep perceptions" (Williams, 2013, p. 10). These words from designer Christopher Williams in his book Origins of form: The shape of natural and manmade things are extremely relevant today.

Applied Mathematics: the Quantitative Liberal Art

If we agree with (Zakaria, 2015) that the core of a liberal arts education is to produce responsible citizens or to provide the training for tomorrow's leaders, we need to identify the essence of leadership qualities, how we can inculcate these through training in a university setting of higher learning, and most importantly in what way applied mathematics is ideally suited as an academic discipline (or at least a key academic discipline) to provide that training. What, then, are the most important leadership qualities? Moreover, are these fixed in time or could new skills gain importance in a new era? Below, we argue firstly that a core set of competencies based on a strong grounding in the skills of the trivium has always been and remains important. Secondly, we argue that in planning the training of tomorrow's leaders we must also acknowledge the ever-increasing importance of a deep understanding and comfort with scientific and computational knowledge as developed in a modern quadrivium. Finally, we argue that applied mathematics not only is central to the development and flourishing of these more advanced and future-oriented leadership skills, but also acts as a reinforcement of the traditional leadership skills which have served world leaders for millennia.

A quick search of the background of world leaders reveals that, for example, many US presidents are trained as lawyers including Abraham Lincoln. This is understandable as law is a second degree and most of the students have already received training in a particular subject area before entering law school. Once there, they are further trained how to think logically, present airtight arguments, often from two opposing sides, in addition to professional training. These are again in alignment with the trivium part of a liberal arts education (namely grammar, rhetoric, and dialectic). A good leader must be knowledgeable and exert confidence. They must be able to think logically, formulate their own arguments, rebut those of others, and speak persuasively. These skills would have been recognized as crucial by the Ancient Greeks and remain as important today. This is one reason why a liberal arts education has endured for as long as it has.

What are the qualities of tomorrow's citizens who can lead us into the unknown future? In addition to those we listed above, which will surely remain important, additional leadership qualities are rising in importance as we enter a new era in which technical competence in the sciences and computational thinking become as important as communication skills, if not more so. Thus, tomorrow's leaders must be knowledgeable about and comfortable with the technical aspects of our daily life in order to lead in a highly competitive environment driven internationally by the accelerating development of science and technology. Moreover, by science and technology which inform or shape more and more aspects of the daily lives of the citizenry. We believe that applied mathematics is one of the programmes that can bring to fullness the liberal arts education by providing both the technical training of the quadrivium and the communication training of the trivium in one place. On the other hand, one could also argue every scientific discipline can provide a rigorous discipline specific training and what we need is to add liberal arts education to its curriculum.

Before we make a case for applied mathematics, we wish to introduce a simile to guide the reader from an arts background. We all know that the leader of an orchestra is the conductor. It may be beneficial to examine what is the typical training received by a conductor. In the majority of the cases, a conductor usually knows how to play one or two music instruments, and most often it is the piano. In addition, a conductor must receive training in other areas including orchestration to understand the roles of other instruments in an orchestra. Of course, a conductor could be an expert in violin or other instruments. However, the piano is preferred as it is relatively easy for a beginner and covers the widest range of keys and moreover is the most versatile. In addition, training in piano lays a solid foundation for many music related careers such as music therapy, music education, composition and of course conducting in an orchestra. In this sense, applied mathematics is like the piano of science and other scientific disciplines are like more specialized instruments.

Below, we will provide detailed arguments and make the case for applied mathematics as the lynchpin of the liberal arts, including what already exists and what new ingredients are needed in a modern quantitative liberal arts education. To begin, we note that the hallmarks of liberal arts education are its inherent interdisciplinarity and emphasis on breadth, which are shared by applied mathematics.

Interdisciplinarity

The heart of applied mathematics is interdisciplinarity. The reason for this is due to the nature of applied mathematics. It has been said that "mathematics becomes 'applied' when it is used to solve real-world problems" (Higham et al., 2015, p.1), and that "Applying mathematics means using a mathematical technique to derive an answer to a question posed from outside mathematics" (Pedley, 2005, as cited in Wilson, 2014, P.176). In other words, applied mathematics is an inherently outward-looking subject which only functions in connection and collaboration with the knowledge and expertise of people in other disciplines. As such, in addition to mathematical training, an applied mathematics major will often choose an area of application of mathematics. The traditional areas include other scientific disciplines such as physics (astronomy), chemistry, and the biological sciences, as well as engineering fields. Relatively new areas of application include economics and finance, medical and health sciences, and increasingly the social sciences and humanities subjects.

Obviously, it is not possible or necessary to put the entire curriculum of another scientific discipline into the applied mathematics program except in the case of double majors. A minor program or simply several courses as free electives will often suffice. The main point is that even a limited exposure to a subjective or scientific discipline, properly structured and design, could provide enough motivation for a life-long career. To be able to pursue a career outside mathematics is one of the main advantages of an applied mathematics education. It can be done in a variety of ways, including a postgraduate training in the application areas, after being exposed to those areas as an applied mathematics major. A training in applied mathematics also provides the option to pursue a different career different from one's first job, by circumstances or as a personal choice.

Moreover, the interdisciplinary nature of applied mathematics gives a further liberal arts skill to the applied mathematics undergraduate; namely, they must be able to rapidly learn from a different discipline in order to apply mathematics to it. While they will not become experts in that second discipline, they will need to rapidly assimilate the essentials. And they can repeat this as they work on different applications. This kind of flexibility of mind is predicted to be increasingly important in a world which will increasingly be in flux. Today's students will be working in careers tomorrow which do not even exist today.

Breadth and Depth

The breadth in applied mathematics comes from its mathematical, computational, and interdisciplinary training. As in any academic program, how to achieve the balance between breadth and depth in applied mathematics is of critical importance. We argue that balance is not simply counting the credits and the number of hours spent on specific courses. It should be measured with learning outcomes. If our aim is to produce “tomorrow leaders” as liberal arts education strives for, we need to think carefully about what constitutes the key set of leadership qualities and design the curriculum accordingly.

The ability to think logically is obviously important. The ability to lead a team and communicate effectively among team members as well as with people outside your team is also critical. The ability to solve a technical problem or in-depth technical training, on the other hand, is less important in this context. It is not the objective of the program to teach all the technical details in a subfield of mathematics. On the contrary, an applied mathematics major should be able to learn the required materials on a “just-in-time” basis. Indeed, in order to acquire domain-specific knowledge from outside of mathematics, students will have to work closely with experts in that field. Thus, they must develop key interpersonal and communication skills central to leadership. Therefore, a liberal arts-oriented program such as applied mathematics ideally will provide training to individuals who are technically capable to the extent that she or he can use their analytical and communication skills to lead a team with a diverse background to identify non-mathematical problems and solve them, in the real world.

Problem Solving Skills

The main objective of applied mathematics training is to teach problem solving skills, which includes identifying problems in the first place. Finding problems sometimes is the most important part of solving problems. Within applied mathematics, much respect is given to mathematicians who are good at identifying rich problems. This is the reason that modeling and interdisciplinary training has become an integral part of any applied mathematics program. We emphasize that this training is not technical-oriented even though learning specific techniques is an important part of the training. The training on identifying problems, developing and simplifying models, and then solving them using analytical or computational means (or, more likely, both) so that useful insights can be gained is the essence of problem-solving skills in applied mathematics-and beyond. In many cases, physical and modelling intuitions are as important as technical abilities in the first few steps, i.e., identifying problems, developing, and simplifying models, which is the reason that modeling is often called an art instead of a science! A successful program should encourage a passion for discovery and a deep intellectual curiosity for the world, so that a limitless source of questions is generated by the meeting of important applications and the student's keen curiosity.

Emotional Intelligence

Nobel laureate Richard Feynman remarked that there is a general perception that when a flower is presented to a scientist, only the inner beauty matters, which is of course not the case. As a physicist, he argued, both the inner and outer beauties are to be appreciated (Feynman & Leighton, 1985). In other words, if a physics Nobel laureate claims that the very aim of arguably the hardest of the hard sciences is a union of inner and outer aesthetics, then the perception that STEM majors lack emotional intelligence may be unfounded. So, from where does this perception come? The problem may well be that some STEM students lack an understanding of human behaviour and communication skills. Therefore, exposure to basic subjects in social science and the humanities such as economics, sociology, and psychology is beneficial to develop basic understanding of human behaviour. Developing communication skills is also at the heart of the trivium, and is discussed later. Of course, not everything needs to be taught in classrooms. Extracurricular activities such as voluntary services or simply by interacting with students in social science and humanities could be other effective means to develop emotional intelligence.

Communication Skills

Communication skills are not typically emphasized in applied mathematics programs, especially in the more traditional programs where technical skills are often considered to be the priority. On the other hand, feedback from employers of STEM students has been loud and clear. The career trajectories of students with STEM and humanity and social sciences background have also clearly demonstrated the importance of communication skills including reading, writing, and presenting key messages to stack-holders and the ability to work in a team environment. Indeed, despite STEM majors having higher starting salaries and being more immediately employable, data shows that humanities majors can expect to earn the same total salaries over their careers. In other worse, despite the difficulties faced by humanities and social sciences graduates of landing a job in the first place when compared to STEM students, they often do better in their careers in terms of promotions and taking up leadership positions inside the same organizations. This is attributed to their ability to effectively communicate their ideas and work as productive team members. Therefore, any successful applied mathematics program must emphasize the importance of providing training in communication skills, by including group projects, presentations, and technical writing as a core part of its curriculum.

Quantitative Liberal Arts

When one of us (HH) was preparing a talk about applied mathematics for high school and undergraduate students, we came across the website for the undergraduate applied mathematics program at Harvard, which declared that applied mathematics is the quantitative liberal art. We could not agree more, and indeed our growing conviction is that this is the best description of applied mathematics. We have argued herein that the ideals of liberal arts education are truly reflected in a well-structured applied mathematics program. Such a program provides the breadth, depth, and interdisciplinarity a liberal arts education requires. It teaches communication skills, emotional intelligence, and problem solving. And all the quantitative and computational skills the modern and future leadership space requires.

The distinction between a (quantitative) liberal arts education and technical training can also find parallel in the following comparison. An applied mathematician may work on the same problem as a structural engineer and apply similar methods to solve the technical problems involved in building a bridge, for example. The difference between an applied mathematician and a structural engineer is that the applied mathematician is also interested in understanding the general principles and extending the methodology so that they can be used for problems in many other industries, or even in areas that are completely unrelated to civil engineering, while the engineer would focus on improving the method for building stronger and better bridges or other structures.

As an example, we take the method named after French mathematician Jean-Baptiste Joseph Fourier (1768-1830). The Fourier series was developed by him to represent the temperature when he was investigating the heat transfer problem – a very specific and limited application. However, after further investigations by himself as well as by other (applied) mathematicians, the method and its variations now enjoy success in a wide range of other areas including signal processing, acoustics, and optics with application in revolutionizing the music recording industry and other entertainment industries, as well as much modern engineering.

As a related illustration, the heat equation Fourier derived turned out to be mathematically equivalent to the celebrated Black-Scholes equation for European options in finance.

Applied Mathematics in the New (Data) Era

Applied mathematics must evolve as for any academic discipline. As the amount of data explodes, our world is becoming increasingly a data-driven one. Therefore, we need to revise our curriculum to meet the challenges of a big data world.

Curriculum Reform

There has been a quiet switch in the curriculum of applied mathematics as computers become more powerful and accessible and sophisticated computational techniques are being developed and improved. Analytical techniques are supplemented with computational approaches and computational simulations are used more and more in the place of "experimental evidence". As a result, courses on computer programming and computational methodologies have become standard in many applied mathematics programs. However, such training is insufficient to meet the industrial and societal needs for highly skilled individuals who will lead the digital revolution in the 21st century. The breathtaking advances in machine learning due to huge successes of deep learning methodologies have captured the imagination of the general public as well as a broad sector of industry, and the scientific community. This poses both opportunities and challenges in reforming the applied mathematics curriculum. The boundaries between many traditional scientific disciplines have blurred. For example, data science has emerged as a darling of many forward-looking academic institutions, if not all. The questions are: what is the role of applied mathematics in this data-driven future and how should it respond to the digital revolution?

First of all, the critical role that applied mathematics is playing in the development of data science, along with computer science and statistics, mirrors the centrality of applied mathematics in the creation of computer science many years ago. Alan Turing was an (applied) mathematician and the father of modern computer science. In addition to essentially inventing computer science and using his invention to save millions of lives in the Second World War, his reaction-diffusion model has been used to study pattern formation in animals, e.g., stripes on zebra or whiskers in mice. Another pioneer of modern computer science is John von

Neumann, an applied mathematics genius who laid the foundation for game theory in economics as well as building one of the first functional electronic computers in the basement of the Institute for Advanced Studies in Princeton. Indeed, many prominent computer scientists were also mathematicians.

But what about applied mathematics as an academic discipline itself? Perhaps it is easier to illustrate how the discipline of applied mathematics could respond by an example. Asymptotic analysis is an old course in many traditional applied mathematics programs that has all but disappeared from the curriculum. Despite being extremely useful, it is difficult for many students to learn, as mastering it requires deep understanding of the problem at hand and good intuitions – which is why it is often referred to as an art! However, the idea behind asymptotic analysis is very powerful, which can be valuable even in the new data era. The idea is also simple. Many problems may seem to be complicated and hopeless, but underneath the surface there exists a simple relation between two main forces. If we can find that relationship, then many problems can be simplified and solved, at least under certain conditions. Asymptotic analysis can help us to find that hidden relationship by reviewing the relative size of various terms in a complicated mathematical equation. But many problems even after simplifications still cannot be solved, which limits the power of the method. What if we combine asymptotic analysis with new computer algorithms? How about if we are looking for hidden relationships from the data directly? This is exactly what a new approach called model reduction aims for.

Obviously, we need to revise our computational methods course and perhaps even the way our foundational courses such as linear algebra are taught. We should introduce newly developed machine learning techniques including deep learning into our curriculum. More importantly, we believe, is that more statistical thinking (in addition to computational skills) should be incorporated into the main applied mathematics curriculum. Applied mathematics majors should be trained so that they know how to make decisions in the face of uncertainties, a skill which has been ignored in many applied mathematics programs. Therefore, in addition to introductory statistics and probability courses, we need to expose applied mathematics students to computational methods using more advanced probability and statistical methods. Monte-Carlo and stochastic models should be introduced either in the computational methods courses or in the modeling courses.

Digital Social Science and Humanity

In the spring of 2019, the Fields Institute for Research in Mathematical Sciences (Fields Institute), a world-renowned mathematics research center in Toronto, organized for the first time a six-month interdisciplinary thematic program (Fields Institute, 2019). Among many conferences, workshops, and graduate courses held in different areas of mathematics, there was also a one-week workshop on digital humanities with two historians as lead-organizers. A follow-up workshop was planned but postponed due to the global COVID-19 pandemic. There have been similar programs and workshops in other parts of the world and this trend is almost certain to continue. The reason is the need for more quantitative techniques to study phenomena which have been out of reach due to either a lack of data or of adequate methodologies; both are changing rapidly. At Beijing Normal University-Hong Kong Baptist University United International College (UIC), a new digital social science program has just been launched with a joint effort from social scientists and computational scientists (UIC, n.d.). As the program matures, we believe that joint supervision of students by applied mathematicians and social scientists will become a norm of new interdisciplinary programs.

A Case Study

In the Appendix A, we have provided some examples of the courses offered by the Harvard program. The Harvard curriculum consists of courses in statistics, probability, data science, and machine learning. In the Appendix B, we have also provided the curriculum of the newly developed applied mathematics program at UIC.

Curriculum

As seen from the list of courses, the UIC program is similar to the one offered by Harvard with some differences in individual courses. An applied mathematics major will need to take foundational courses in mathematics, such as calculus and linear algebra, and in many programs programming languages and other computer science courses. Courses such as introductory probability, statistics, and (numerical) analysis are often included as the basic core. At a higher level, (ordinary and partial) differential equations are usually required while optimizations, advanced numerical methods, and other courses may be optional, varying among programs.

As in the Harvard program, the UIC curriculum also consists of courses in statistics, probability, data science, and machine learning. What is not seen on the list explicitly is the liberal arts component, which is covered by the University Core, General Education courses, and the courses offered by the Whole Person Education Office, including emotional intelligence and voluntary services.

Challenges

When the UIC program was being developed, the challenge was how to maintain the balance between breadth and depth. Going forward, the main challenge is how to constantly revise the curriculum so that newly developed methodologies and subjects such as the ones discussed in the previous sections can be incorporated into the curriculum. Furthermore, even with a well-developed curriculum, how to guide individual students to find a path that is most suitable for them remains the most important challenge.

As any program, UIC applied mathematics is limited in its course selections. For example, the current curriculum does not include exposure to economics, psychology, and other social science subjects, nor does it include music education, which was one of the original skills included in the liberal arts education in ancient Greece. One of the challenges ahead is how to give individual students access while not overwhelming them with too much course work.

Conclusion

Before we end this article, we wish to point out that mathematics was developed due to both external needs (especially in its early days) and human curiosity. As in the development of any scientific discipline, both are needed for moving applied mathematics into the new era.

Problem-Driven or Curiosity-Driven

Problems from the real world will continue to provide motivations and inspirations needed for developing new methods and techniques. Differential equations played a central role so far in the development of applied mathematics as a discipline. They are usually models based on physical laws (first principles) represented by mathematical quantities involving the derivatives of unknown functions. Solving differential equations and finding their properties have occupied many of the great minds in modern science and mathematics. The solutions to many differential equations remain out of reach despite great efforts from generations of mathematicians. Not all the questions have direct real-world applications and some of them are out of pure curiosity. For example, differential equations originating from applications in engineering are usually limited in three spatial dimensions. However, mathematicians and sometimes applied mathematicians are interested in the properties of differential equations in many dimensions (more than three or four), way before people found real applications, e.g., in finance or other areas.

The new era we have entered is data-driven, which requires new tools to be developed. On the other hand, newly developed tools and methodologies have new properties that are not well-understood. This will create huge demand for highly qualified, quantitative-oriented individuals including applied mathematics majors. Like all liberal arts students, applied mathematics majors are strongly encouraged to stay curious as curiosity is the source of all enquiries and applied mathematics is no exception. An excellent example is given by the late Joe Keller, an eminent Stanford applied mathematician who is well known for his curiosity-driven research, including a paper on the swing of ponytails, prompted by his observations of the sideway motion of the ponytails resulting from vertical head-motions of a student runner on campus (Keller, 1957, 2010).

Applied Mathematics and Applications of Mathematics

It is probably worth pointing out that applied mathematics is not the same as applications of mathematics. Mathematics has been applied in many areas especially in physical sciences and engineering as well as economics. Applied mathematics as a scientific discipline is much more than the applications of mathematics, which happens in one direction. Applied mathematics, as we discussed earlier, is both problem- and curiosity-driven. In addition, applied mathematicians are interested in the extension and generalization of mathematical models and techniques developed for solving specific problems so that new mathematical tools and theories can be developed and applied to other areas.

In this article, we have shown how applied mathematics and liberal arts education share many common features, which explains why Harvard has claimed its applied mathematics program is one in the "quantitative liberal arts". If we can summarize it in one sentence, perhaps we can borrow it from Drew Faust, former president of Harvard, a liberal arts education (applied mathematics) teaches students the skills "that will help them to get ready for their [second, third, fourth, fifth and] sixth job, not [only] their first job" (Zaharia, 2015). It is worth emphasizing that these skills are not about being able to carry out complicated calculations. It is about developing the ability to reason and solve non-mathematical problems. Abraham Lincoln, known for his superb statesmanship, credited geometry for his success in developing the skills for success in law school and politics (Ellenberg, 2021). He was struggling to understand how one can demonstrate beyond reasonable doubt about certain cases and the inspiration came from the proofs in geometry, which only require axioms (everyone can accept) and logic, independent of subjective rules (many would object).

Clearly, there are still many questions which need to be answered. We are hoping that these questions and this article can motivate colleagues from other universities including applied mathematics programs to work together in making applied mathematics the best liberal arts education we can offer.

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Appendix A
Harvard Applied Mathematics Program

Here we list some of the courses from the Harvard website:
https://www.seas.harvard.edu/applied-mathematics/courses

1. APMTH 10. Computing with Python for Scientists and Engineers
2. APMTH 22A. Solving and Optimizing.
3. APMTH 22B. Integrating and Approximating.
4. APMTH 50. Introduction to Applied Mathematics.
5. APMTH 91R. Supervised Reading and Research.
6. APMTH 99R. Thesis Research.
7. APMTH 101. Statistical Inference for Scientists and Engineers.
8. APMTH 104. Complex and Fourier Analysis with Applications to Science and Engineering.
9. APMTH 105. Ordinary and Partial Differential Equations.
10. APMTH 107. Graph Theory and Combinatorics.
11. APMTH 108. Nonlinear Dynamical Systems.
12. APMTH 111. Introduction to Scientific Computing.
13. APMTH 115. Mathematical Modeling.
14. APMTH 120. Applied Linear Algebra and Big Data.
15. APMTH 121. Introduction to Optimization: Models and Methods.
16. APMTH 122. Convex Optimization and Its Applications.
17. APMTH 201. Physical Mathematics I.
18. APMTH 203. Introduction to Disordered Systems and Stochastic Processes.
19. APMTH 205. Advanced Scientific Computing: Numerical Methods.
20. APMTH 207. Advanced Scientific Computing: Stochastic Methods for Data Analysis, Inference and Optimization.
21. APMTH 211. Models, Algorithms and Data.
22. APMTH 216. Inverse Problems in Science and Engineering.
23. APMTH 226. Neural Computing.
24. APMTH 233. Interplay between Control and Learning.
25. APMTH 299R. Special Topics in Applied Mathematics.

Appendix B
BNU-HKBU UIC Applied Mathematics Program

Here we list the major required and major elective courses:
https://dst.uic.edu.cn/am_en/students/undergraduate_studies/curriculum.htm

1. COMP1023. Foundations of C Programming
2. COMP2003. Data Structures and Algorithms
3. MATH1053. Linear Algebra I
4. MATH1063. Linear Algebra II
5. MATH1073. Calculus I
6. MATH1083. Calculus II
7. MATH2043. Ordinary Differential Equations
8. MATH2053. Mathematical Analysis
9. MATH2063. Probability and Statistics
10. MATH3033. Partial Differential Equations
11. MATH3163. Real Analysis
12. MATH4083. Numerical Analysis
13. MATH4093. Complex Analysis
14. MATH4103. Mathematical Modelling
15. MATH4123. Final Year Project I (MATH)
16. OR4023. Optimization
17. STAT3073. Statistical Computing
18. BIOL2003. General Biology
19. COMP3013. Database Management Systems
20. DS4023. Machine Learning
21. MATH2013. Introduction to Mathematical Finance
22. MATH3013. Discrete Mathematics
23. MATH3083. Markov Chain and Queuing Theory
24. MATH3093. Supply Chain Modelling
25. MATH3143. Differential Geometry
26. MATH3173. Applied Stochastic Process
27. MATH4003. Graph Theory
28. MATH4033. Computational Finance

29. MATH4113. Selected Topics in Applied Analysis
30. MATH4143. Functional Analysis
31. MATH4153. Numerical Methods for Differential Equations
32. MATH4163. Final Year Project II (MATH)
33. OR3013. Linear Programming and Integer Programming
34. PHYS2003. Principles of Physics
35. STAT4013. Multivariate Analysis
36. STAT4073. Data Mining

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