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Convergence in Wavelet Collocation Methods for Parabolic Problems

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Abstract. This paper studies the second-generation interpolating wavelet collocation methods in space and different Euler time stepping methods for parabolic problems. The convergence and stability are investigated. The operators are formulated using an efficient and exact formulation. The numerical results verify the efficiency of the methods.

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1 Introduction

Even though much effort has been made in designing wavelet collocation methods for solving partial differential equations (see [1–7] etc.),however, there has been no rigorous study on stability and convergence. In [8],we studied adaptive wavelet collocation methods in solving partial integro-differential equations for option pricing in a market driven by jump-diffusion processes. In this paper, we consider Problem 1.1 with the second-generation wavelet collocation methods and address the convergence analysis.

Problem 1.1. Let $\Omega=(0,1)$. Find $u(x,t)\in C^1((0,T],H^2(\Omega))$ such that

$$\begin{aligned} &\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, \quad \text{in} \quad \Omega\times(0,T], \\ &u(0,t)=u(1,t)=0, \quad u(x,0)=u_0(x). \end{aligned} \tag{1.1}$$

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The second generation interpolating wavelet is very convenient and popular in solving PDEs, we use a hybrid of a wavelet collocation method in space and a Euler time stepping (implicit or Crank-Nicolson) to discretize Problem 1.1. We first formulate the operators exactly and efficiently in the wavelet spaces. Then we perform convergence analysis for the numerical schemes. Finally we verify the convergence and demonstrate the efficiency through numerical experiments.

This paper is organized as follows. Section 2 introduces the exact and efficient formulation of operators in a wavelet collocation setting. Section 3 introduces the continuous time collocation method. Section 4 analyzes numerical schemes. Section 5 presents the numerical results.

2 Second generation interpolating wavelets

2.1 Wavelet on an interval

Consider the interval $\Omega=[0,1]$. For each level j, we place a grid

$$\mathcal{G}^j=\left\{x_{j,k}\,\middle|\,x_{j,k}=\frac{k}{2^j},k=0,1,\cdots,2^j\right\}$$

on Ω. A set of interpolating scaling functions $\{\phi_{j,k},k=0,1,\cdots,2^j\}$ can be constructed using the interpolating subdivision scheme and they satisfy the two-scale relationship

$$\phi_{j,k}=\sum_{l=0}^{2^{j+1}}h_{j,k,l}\phi_{j+1,l} \tag{2.1}$$

where

$$h_{j,k,l}=\begin{cases}\delta_{2k-l}, & \text{for}\quad l=0,2,\cdots,2^{j+1},\\ q_k^j(x_{j+1,l}), & \text{for}\quad l=1,3,\cdots,2^{j+1}-1,\end{cases} \tag{2.2}$$

and $q_k^j(x)$ is the Lagrange interpolating polynomial through the p points closest to $x_{j,k}$ on $\mathcal{G}^j$.The scaling function space V_j:=span$\{\phi_{j,k}(x),k=0,1,\cdots,2^j\}$ satisfies a second-generation multiresolution analysis. We denote the filter h in Equation (2.1) in the matrix form:

$$H_j=E^j+\tilde{S}^jD^j$$

where

$$\begin{aligned}&E^j_{k,m}=\delta_{2k-m}, && k=0,1,\cdots,2^j,m=0,1,\cdots,2^{j+1},\\ &D^j_{k,m}=\delta_{2k+1-m}, && k=0,1,\cdots,2^j-1,m=0,1,\cdots,2^{j+1},\\ &\tilde{S}^j_{k,l}=q^j_k(x_{j+1,2l+1}), && k=0,1,\cdots,2^j,l=0,1,\cdots,2^j-1.\end{aligned} \tag{2.3}$$