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Global Existence and Asymptotic Stability for the Initial Boundary Value Problem of the Linear Bresse System with a Time-Varying Delay Term

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Abstract. The authors of this paper study the Bresse system in bounded domain with delay terms. First, we prove the global existence of its solutions in Sobolev spaces by means of semigroup theory. Furthermore, the asymptotic stability is given by using an appropriate Lyapunov functional.

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1 Introduction

In this paper, we investigate the existence and decay properties of solutions for the initial boundary value problem of the linear Bresse system of the type

$$\begin{cases} \rho_1\varphi_{tt}-k_1(\varphi_x+\psi+lw)_x-lk_3(w_x-l\varphi)+\mu_1\varphi_t \\ \qquad +\mu_2\varphi_t(x,t-\tau_1(t))=0, \\ \rho_2\psi_{tt}-k_2\psi_{xx}+k_1(\varphi_x+\psi+lw)+\tilde{\mu_1}\psi_t \\ \qquad +\tilde{\mu_2}\psi_t(x,t-\tau_2(t))=0, \\ \rho_1 w_{tt}-k_3(w_x-l\varphi)_x+lk_1(\varphi_x+\psi+lw)+\tilde{\tilde{\mu_1}}w_t \\ \qquad +\tilde{\tilde{\mu_2}}w_t(x,t-\tau_3(t))=0, \end{cases} \tag{1.1}$$

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where $(x,t)\in(0,L)\times(0,\infty)$. Moreover, $\tau_i(t)>0$ $(i=1,2,3)$ is a time delay and $\mu_1,\mu_2,\tilde{\mu_1},\tilde{\mu_2}$, $\tilde{\tilde{\mu_1}},\tilde{\tilde{\mu_2}}$ are positive real numbers. This system is subject to the following Dirichlet boundary conditions

$$\begin{cases} \varphi(0,t)=\psi(0,t)=w(0,t)=0, \\ m\varphi_{tt}(L,t)+\gamma\varphi_t(L,t)+k_1(\varphi_x(L,t)+\psi(L,t)+lw(L,t))=0, \\ I_m\psi_{tt}(L,t)+\tilde{\gamma}\psi_t(L,t)+k_2\psi_x(L,t)=0, \\ mw_{tt}(L,t)+\tilde{\tilde{\gamma}}w_t(L,t)+k_3(w_x(L,t)-l\varphi(L,t))=0, \end{cases}$$

and to the following initial conditions

$$\begin{cases} \varphi(x,-t)=\varphi_0(x),\;\; \varphi_t(x,0)=\varphi_1(x),\;\; \psi(x,-t)=\psi_0(x), & x\in(0,L), \\ \psi_t(x,0)=\psi_1(x),\;\; w(x,-t)=w_0(x),\;\; w_t(x,0)=w_1(x), & x\in(x,L), \\ \varphi_t(x,t-\tau_1(t))=f_0(x,t-\tau_1(t)), \qquad \text{in } (0,L)\times[0,\tau_1(0)], \\ \psi_t(x,t-\tau_2(t))=f_0(x,t-\tau_2(t)), \qquad \text{in } (0,L)\times[0,\tau_2(0)], \\ w_t(x,t-\tau_3(t))=f_0(x,t-\tau_3(t)), \qquad \text{in } (0,L)\times[0,\tau_3(0)]. \end{cases}$$

The initial data $(\varphi_0,\varphi_1,\psi_0,\psi_1,w_0,w_1,f_0,\tilde{f_0},\tilde{\tilde{f_0}})$ belong to suitable Sobolev space. By w,ψ and φ we are denoting the longitudinal, vertical and shear angle displacements. The original Bresse system is given by the following equations (see [1]).

$$\begin{cases} \rho_1\varphi_{tt}=Q_x+lN+F_1, \\ \rho_2\psi_{tt}=M_x-Q+F_2, \\ \rho_1 w_{tt}=N_x-lQ+F_3, \end{cases}$$

where we use N,Q and M to denote the axial force, the shear force and the bending moment respectively. These forces are stress-strain relations for elastic behavior and given by

$$N=Eh(w_x-l\varphi), \quad Q=Gh(\varphi_x+\psi+lw) \quad \text{and} \quad M=EI\psi_{xx},$$

where G, E, I and h are positive constants. Finally, by the terms F_i we are denoting external forces. The Bresse system without delay (i.e $\mu_2=\tilde{\mu_2}=\tilde{\tilde{\mu_2}}=0$) is more general than the well-known Timoshenko system where the longitudinal displacement s is not considered $(l=0)$. There are a number of publications concerning the stabilization of Timoshenko system with different kinds of damping (see [2–5]) and the references therein. A general decay result, from which the exponential and polynomial rates of decay are only special cases, was also established by Kafini [6]. Raposo and al [4] proved the exponential decay of the solution for the following linear system of Timoshenko-type beam equations with linear frictional dissipative terms

$$\begin{cases} \rho_1\varphi_{tt}-Gh(\varphi_x+\psi+lw)_x-lEh(w_x-l\varphi)+\mu_1\varphi_t=0, \\ \rho_2\psi_{tt}-EI\psi_{xx}+Gh(\varphi_x+\psi+lw)+\tilde{\mu_1}\psi_t=0. \end{cases}$$