Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **11**, No. 6, pp. 1376-1397

DOI: 10.4208/aamm.OA-2019-0002
December 2019

# Two-Level Hierarchical PCG Methods for the Quadratic FEM Discretizations of 2D Concrete Aggregate Models

Yingxiong Xiao*, Heng Chen and Lingjie Xie

*Civil Engineering and Mechanics College, Xiangtan University, Xiangtan 411105, Hunan, China*

Received 3 January 2019; Accepted (in revised version) 21 March 2019

**Abstract.** The concrete aggregate model is considered as a type of weakly discontinuous problem consisting of three phases: aggregates which randomly distributed in different shapes, cement paste and internal transition zone (ITZ). Because of different shapes of aggregate and thin ITZs, a huge number of elements are often used in the finite element (FEM) analysis. In order to ensure the accuracy of the numerical solutions near the interfaces, we need to use higher-order elements. The widely used FEM softwares such as ANSYS and ABAQUS all provide the option of quadratic elements. However, they have much higher computational complexity than the linear elements. The corresponding coefficient matrix of the system of equations is a highly ill-conditioned matrix due to the large difference between three phase materials, and the convergence rate of the commonly used solving methods will deteriorate. In this paper, two types of simple and efficient preconditioners are proposed for the system of equations of the concrete aggregate models on unstructured triangle meshes by using the resulting hierarchical structure and the properties of the diagonal block matrices. The main computational cost of these preconditioners is how to efficiently solve the system of equations by using linear elements, and thus we can provide some efficient and robust solvers by calling the existing geometric-based algebraic multigrid (GAMG) methods. Since the hierarchical basis functions are used, we need not present those algebraic criterions to judge the relationships between the unknown variables and the geometric node types, and the grid transfer operators are also trivial. This makes it easy to find the linear element matrix derived directly from the fine level matrix, and thus the overall efficiency is greatly improved. The numerical results have verified the efficiency of the resulting preconditioned conjugate gradient (PCG) methods which are applied to the solution of several typical aggregate models.

**AMS subject classifications**: 65N55, 65N22

**Key words**: Concrete aggregate model, hierarchical quadratic elements, ill-conditioned matrix, algebraic multigrid method, preconditioner.

*Corresponding author.
*Email:* xyx610xyx@xtu.edu.cn (Y. X. Xiao)

 

## 1 Introduction

Concrete is considered as a type of composite consisting of three phases: cement paste, coarse aggregate and interfacial transition zone (ITZ) on meso-level. The macroscopic mechanical properties and failure modes of concrete materials are mainly determined by the properties of component materials, the volume content and mixture ratio of aggregates considered [1]. Therefore, it is necessary to establish appropriate mesoscopic models in order to perform some numerical analysis. There are many methods for establishing random geometric models of concrete, see references such as [2–6]. Recently, a hybrid realization method is proposed in order to rapidly obtain 3D concrete spherical aggregate models with high volume fraction which can reach about 65% by combining FORTRAN and ANSYS softwares [7]. This method can also easily generate the polyhedron aggregate model with different shapes.

After the geometry of concrete aggregate model is obtained, a huge number of elements are often used in finite element computation because of different shapes of aggregate and thin ITZs. Moreover, in order to ensure the high accuracy of the finite element solution near the interfaces, it is necessary to use higher-order elements, for example, the widely used FEM softwares as ANSYS and ABAQUS all provide the option of quadratic elements. However, higher-order elements need more computer storage space and have much higher computational complexity than the linear elements. Since the large difference between three phase materials in practice, the coefficient matrix of the system of equations is a highly ill-conditioned matrix. The efficiency of the commonly used solvers will be rapidly reduced. Therefore, it is important to design more efficient solving methods to improve the overall efficiency of finite element analysis for the concrete models with random aggregates under the condition of ensuring the accuracy of numerical solution.

Multigrid method has recently been one of the most effective numerical methods for solving the system of equations arising from the finite element discretizations of partial differential equations, in which algebraic multigrid (AMG) method has become the research focus since it has some advantages for solving large-scale system of equations on unstructured grids [8–12]. If we can use the known information that is easily available in most finite element applications, for instance, the type of the partial differential equation considered, the number of physical unknowns residing in each grid and nodal coordinates on the finest grid level, more robust and efficient AMG as the geometric-based AMG (GAMG) methods can be obtained. There are quite a lot of research results, see for example AMGe method [13, 14] and AMG method based on element agglomeration scheme [15]. Several GAMG methods were designed and analyzed for some different problems in references [16–18]. We would like to refer readers to [20–23] for the recent efforts to apply AMG methods to the system of equations of low-order finite elements for elasticity problems. Recently, some better research results on GAMG methods have been developed in [24–26] for higher-order discretizations of linear elasticity. Hierarchical elements are often used in designing fast solving algorithms and this can help to reduce the