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Bifurcation Analysis for a Free Boundary Problem Modeling Growth of Solid Tumor with Inhibitors

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Abstract: This paper is concerned with the bifurcation analysis for a free boundary problem modeling the growth of solid tumor with inhibitors. In this problem, surface tension coefficient plays the role of bifurcation parameter, it is proved that there exists a sequence of the nonradially stationary solutions bifurcate from the radially symmetric stationary solutions. Our results indicate that the tumor grown in vivo may have various shapes. In particular, a tumor with an inhibitor is associated with the growth of protrusions.

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1 Introduction

During the past forty years, a number of mathematical models have been studied and developed, see review papers [1]–[5] and the references therein. Among those, the growth of solid tumor models, described by partial differential equations with a free boundary, have been given considerable attention, see [6]–[21]. Solid tumor growth can be regarded as a result of various interactions within the micro environment, such as nutrient (e.g. oxygen, glucose), or inhibitors (e.g. inhibitory material developed from the immune system of healthy cells, anti-cancer drugs and radiation administered by medical treatment), etc.

In this paper, we consider a mathematical model describing the stationary state of an

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avascular solid tumor with inhibitor:

$$\Delta\sigma = f(\sigma) \qquad \text{in } \Omega, \tag{1.1}$$

$$\Delta\beta = g(\beta) \qquad \text{in } \Omega, \tag{1.2}$$

$$-\Delta p = h(\sigma,\beta) \qquad \text{in } \Omega, \tag{1.3}$$

$$\sigma = \bar{\sigma} \qquad \text{on } \partial\Omega, \tag{1.4}$$

$$\beta = \bar{\beta} \qquad \text{on } \partial\Omega, \tag{1.5}$$

$$\frac{\partial p}{\partial \nu} = 0 \qquad \text{on } \partial\Omega, \tag{1.6}$$

$$p = \gamma\kappa \qquad \text{on } \partial\Omega. \tag{1.7}$$

In this model, $\Omega \subseteq \mathbf{R}^3$ is the domain occupied by the tumor. σ, β denote the concentration of nutrient and inhibitor within the tumor, respectively. The pressure p within the tumor comes from the proliferation of the tumor cells. $f(\sigma)$, $g(\sigma)$, $h(\sigma,\beta)$ are the nutrient consumption rate, inhibitor consumption rate and tumor-cell proliferation rate function, respectively. $\bar{\sigma}$ and $\bar{\beta}$ are positive constants, $\sigma = \bar{\sigma}$ and $\beta = \bar{\beta}$ mean that the tumor receives constant nutrient and inhibitor supply from the exterior surface, respectively. ν is the outward normal of the free boundary $\partial\Omega$, γ is the surface tension coefficient, and κ is the mean curvature of the free boundary $\partial\Omega$.

According to the medicine and biology, as well as the need of the mathematics, we assume that f, g, h are functions satisfying the following conditions:

(A$_1$) $f \in C^\infty[0,\infty)$, $g \in C^\infty[0,\infty)$, $h \in C^\infty([0,\infty)\times[0,\infty))$;

(A$_2$) $f'(\sigma) > 0$ for $\sigma \geq 0$ and $f(0) = 0$;

(A$_3$) $g'(\beta) > 0$ for $\beta \geq 0$ and $g(0) = 0$;

(A$_4$) $\dfrac{\partial h(\sigma,\beta)}{\partial\sigma} > 0$, $\dfrac{\partial h(\sigma,\beta)}{\partial\beta} < 0$ for $\sigma > 0$, $\beta > 0$ and $h(0,0) < 0$.

$f(\sigma)$ is strictly monotone increasing about σ means the concentration of nutrient is much larger, the tumor cells consume more nutrient in the unit time. $f(0) = 0$ means the nutrient consumption is zero when there is no nutrient, we can make similar explanation for $g(\beta)$. $h(\sigma,\beta)$ is strictly monotone increasing about σ and decreasing about β means increasing the concentration of the nutrient and inhibitor will enlarge and lower the proliferation rate of the tumor cells, respectively. $h(0,0) < 0$ means that the number of tumor cells decreases when the concentration of nutrient and inhibitor are all zero. Obviously, these assumptions satisfy the medicine and biology principle.

For the system (1.1)–(1.7) without inhibitors, i.e., $\beta = 0$, the authors of [13], [15] and [16] studied the linear case: $f(\sigma) = \sigma$, $h(\sigma) = \mu(\sigma - \widetilde{\sigma})$, and proved the existence of a unique radially symmetric solution and a sequence of nonradially stationary solutions for this system in two-dimensional case and three-dimensional case respectively. In [11], the above results were extended to general case with $f(\sigma)$, $h(\sigma)$ are smooth functions. For the case $\beta \neq 0$, the existence of radially symmetric stationary solutions and nonradially stationary solutions were analysed for the linear case of (1.1)–(1.7) by Cui *et al.* in [10], [12], [20] and [21]. For general case of (1.1)–(1.7), the existence of the radially symmetric solutions was studied by