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TACKLING INDUSTRIAL-SCALE SUPPLY CHAIN PROBLEMS BY MIXED-INTEGER PROGRAMMING*

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Abstract

The modeling flexibility and the optimality guarantees provided by mixed-integer programming greatly aid the design of robust and future-proof decision support systems. The complexity of industrial-scale supply chain optimization, however, often poses limits to the application of general mixed-integer programming solvers. In this paper we describe algorithmic innovations that help to ensure that MIP solver performance matches the complexity of the large supply chain problems and tight time limits encountered in practice. Our computational evaluation is based on a diverse set, modeling real-world scenarios supplied by our industry partner SAP.

Mathematics subject classification: 90B06, 90C05, 90C06, 90C11, 90C90.
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1. Introduction

In the late 1990s, the need for advanced business software to face the challenges of ongoing globalization had become ubiquitous and that need has been continuously increasing ever since then. Thus, various software vendors began to offer so-called advanced planning systems (APS) that helped companies plan and coordinate their growing supply chains and avoid potential bottlenecks in their resources such as labor, material, and machinery.

Many of the underlying questions can naturally be phrased as complex mathematical optimization problems. This article focuses on supply network planning, an optimization task that amounts to the computation of medium- and long-term plans for material procurement, production, transportation, demand fulfillment, stock keeping, and resource capacity utilization over large time horizons and various organizational units such as raw material suppliers, plants, warehouses, and transportation facilities. Some of the quantities may be produced and transported only in discrete lots. The goal is the minimization of the overall business-related costs, which consist of actual costs, e.g., for stock keeping, production, and transportation, as well as penalty costs for missing demand fulfillment [1].

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Mathematically, the resulting optimization problems can be formulated as mixed-integer linear programs (MIPs), which take the general form

$$\min\left\{c^T x : Ax \geq b, x \in \mathbb{Z}^{n_1} \times \mathbb{R}^{n_2}\right\},$$

where $A \in \mathbb{R}^{(n_1+n_2)\times m}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^{n_1+n_2}$. This has many benefits. The expressivity and flexibility of MIP allows to represent many different use cases in one general model. Additionally, solvers provide guarantees on the quality of the supply network plans produced even if the problems are too hard to be solved to optimality. The high feature complexity and level of detail present in real-world supply network problems, however, pose challenges to current state-of-the-art optimization software.

In this article, we describe how these challenges can be addressed even for large-scale real-world supply chain problems in a software product delivered to SAP® users worldwide. In particular, we want to emphasize how the choice for using general MIP solvers as the underlying optimization engine allows to achieve an integrated and general handling of models for different supply chain structures that can easily be adapted and extended for future requirements. On the mathematical side, we describe algorithmic techniques developed inside the academic MIP solver SCIP[1)] [2] in order to improve computational performance on MIP formulations with supply chain structure.

The paper is organized as follows. In Sections 2 and 3, we present the basic MIP models solved and discuss the benefits and challenges of the MIP approach. In Section 4, we explain high-level decomposition techniques that are applied in order to break down large MIP formulations to sizes that can be handled by a general MIP solver within the running time requirements imposed by practitioners. Sections 5 to 8 are dedicated to the algorithmic advances inside the MIP solver SCIP that have been developed to alleviate the computational challenges. In Section 9, we demonstrate their performance impact on a test set derived from diverse real-world supply chain scenarios. Section 10 summarizes our findings.

2. A Mixed-integer Model for Optimal Supply Network Planning

In the following, we detail the base formulation of a mixed-integer model used to optimize medium- or long-term plans for general supply networks including typical supply chain processes such as procurement, production, transportation, and customer-demand fulfillment. This model is an integral part of the Supply Network Planning Optimization (SNP Optimization) function delivered worldwide to users of the SAP® Advanced Planning and Optimization component.[2)] The supply chain plans may cover a time interval of several years and include various organizational units of the supply network (locations) such as raw material suppliers, plants, warehouses, and transportation facilities (see [1] for details). The objective of the model is to minimize the overall business-related costs incurred by stock keeping, production, transport, or missing demand fulfillment. Furthermore, it considers scarce resource capacities required by production and transport activities. Large-scale scenarios may contain up to several thousand products and hundreds of locations.

The basic decisions of the supply chain model to be made are the quantities of material procurement, production, transportation, demand fulfillment, stock keeping, and resource capacity utilization. Some quantities may be produced and transported only in discrete lots. The

1) See http://scip.zib.de/

2) SAP is a registered trademark of SAP SE in Germany and in several other countries.