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Nonlinear Degenerate Anisotropic Elliptic Equations with Variable Exponents and L^1 Data

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Abstract. This paper is devoted to the study of a nonlinear anisotropic elliptic equation with degenerate coercivity, lower order term and L^1 datum in appropriate anisotropic variable exponents Sobolev spaces. We obtain the existence of distributional solutions.

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1 Introduction

In this paper we prove the existence of solutions to the nonlinear anisotropic degenerate elliptic equations with variable exponents, of the type

$$\begin{aligned} &-\sum_{i=1}^{N} D_i a_i(x,u,\nabla u)+g(x,u,\nabla u)=f, && \text{in } \Omega, \\ &u=0, && \text{on } \partial\Omega, \end{aligned} \tag{1.1}$$

where $\Omega\subseteq\mathbb{R}^N$ $(N\geq 3)$ is a bounded domain with smooth boundary $\partial\Omega$ and the right-hand side f in $L^1(\Omega)$, $D_i u=\frac{\partial u}{\partial x_i}$. We suppose that $a_i:\Omega\times\mathbb{R}\times\mathbb{R}^N\to\mathbb{R}$, $i=1,\dots,N$ are

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Carathéodory functions such that for almost every x in Ω and for every $(\sigma,\xi)\in\mathbb{R}\times\mathbb{R}^N$ the following assumptions are satisfied for all $i=1,\dots,N$

$$|a_i(x,\sigma,\xi)|\leq\beta\left(|k(x)|+|\sigma|^{\overline{p}(x)}+\sum_{j=1}^{N}|\xi_j|^{p_j(x)}\right)^{1-\frac{1}{p_i(x)}}, \tag{1.2}$$

$$\sum_{i=1}^{N}(a_i(x,\sigma,\xi)-a_i(x,\sigma,\eta))(\xi_i-\eta_i)>0,\quad \forall\xi\neq\eta, \tag{1.3}$$

$$\sum_{i=1}^{N}a_i(x,\sigma,\xi)\xi_i\geq\alpha\sum_{i=1}^{N}\frac{|\xi_i|^{p_i(x)}}{(1+|\sigma|)^{\gamma_i(x)}}, \tag{1.4}$$

where $\beta>0$, $\alpha>0$, and $k\in L^1(\Omega)$, $\gamma_i:\overline{\Omega}\to\mathbb{R}^+$, $p_i:\overline{\Omega}\to(1,+\infty)$ are continuous functions and $\overline{p}$ is such that

$$\frac{1}{\overline{p}(\cdot)}=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{p_i(\cdot)}.$$

We introduce the function

$$\overline{p}^*(x)=\begin{cases}\frac{N\overline{p}(x)}{N-\overline{p}(x)}, & \text{if } \overline{p}(x)<N,\\ +\infty, & \text{if } \overline{p}(x)\geq N.\end{cases} \tag{1.5}$$

The nonlinear term $g:\Omega\times\mathbb{R}\times\mathbb{R}^N\to\mathbb{R}$ is a Carathéodory function such that for a.e. $x\in\Omega$ and all $(\sigma,\xi)\in\mathbb{R}\times\mathbb{R}^N$, we have

$$|g(x,\sigma,\xi)|\leq b(|\sigma|)\left(c(x)+\sum_{i=1}^{N}|\xi_i|^{p_i(x)}\right), \tag{1.6}$$

$$g(x,\sigma,\xi)\cdot\sigma\geq0, \tag{1.7}$$

where $b:\mathbb{R}^+\to\mathbb{R}^+$ is a continuous and increasing function with finite values, $c\in L^1(\Omega)$ and $\exists\rho>0$ such that:

$$|g(x,\sigma,\xi)|\geq\rho\left(\sum_{i=1}^{N}|\xi_i|^{p_i(x)}\right),\quad \forall\sigma \text{ such that } |\sigma|>\rho. \tag{1.8}$$

In [1], the authors obtain the existence of renormalized and entropy solutions for the nonlinear elliptic equation with degenerate coercivity of the type

$$-\mathrm{div}[a(x,u)|\nabla u|^{p(x)-2}\nabla u]+g(x,u)=f\in L^1(\Omega).$$

For $g\equiv0$ and $f\in L^{m(\cdot)}(\Omega)$, with $m(x)\geq m_-\geq1$, equation of the from (1.1) have been widely studied in [2], where the authors obtain some existence and regularity results for the solutions. If $g\equiv|u|^{s(x)-1}u$,

$$a_i(x,u,\nabla u)=\frac{|D_iu|^{p_i(x)-2}D_iu}{(1+|u|)^{\gamma_i(x)}}$$