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A Weighted Runge-Kutta Discontinuous Galerkin Method for 3D Acoustic and Elastic Wave-Field Modeling

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Abstract. Numerically solving 3D seismic wave equations is a key requirement for forward modeling and inversion. Here, we propose a weighted Runge-Kutta discontinuous Galerkin (WRKDG) method for 3D acoustic and elastic wave-field modeling. For this method, the second-order seismic wave equations in 3D heterogeneous anisotropic media are transformed into a first-order hyperbolic system, and then we use a discontinuous Galerkin (DG) solver based on numerical-flux formulations for spatial discretization. The time discretization is based on an implicit diagonal Runge-Kutta (RK) method and an explicit iterative technique, which avoids solving a large-scale system of linear equations. In the iterative process, we introduce a weighting factor. We investigate the numerical stability criteria of the 3D method in detail for linear and quadratic spatial basis functions. We also present a 3D analysis of numerical dispersion for the full discrete approximation of acoustic equation, which demonstrates that the WRKDG method can efficiently suppress numerical dispersion on coarse grids. Numerical results for several different 3D models including homogeneous and heterogeneous media with isotropic and anisotropic cases show that the 3D WRKDG method can effectively suppress numerical dispersion and provide accurate wave-field information on coarse mesh.

AMS subject classifications: 86-08, 86A15

Key words: Numerical modeling, anisotropy, discontinuous Galerkin method, numerical dispersion, stability.

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1 Introduction

Accurate and efficient numerical method is necessary in computational geophysics for the purpose of resource explorations and understanding the earth. In recent years, there has been a strong interest for the discontinuous Galerkin method (DGM) in seismic numerical simulation. The DGM was first introduced in the early 1970s for solving the linear neutron transport equations [1], and later extended to solve hyperbolic conservation problems by employing discontinuous flux formulations and varies temporal discretizations. The work by Cockburn and Shu [2, 3] formulated the total variation diminishing (TVD) RK discontinuous Galerkin method into a complete mathematical framework. In computational seismology DGM has rapid developments [4–14]. For instance, an arbitrary high-order derivatives DGM proposed by Dumbser and Käser has been widely used in computational geophysics [6–11]. In 3D seismic modeling, the complex geologic geometry and large velocity contrast bring up many challenges to numerical methods due to the large amount of computations and storage requirements. However, the DGMs have been proved successfully to be efficient and high-order accurate in modeling seismic elastic wave propagating in complex media [5,12], even with viscoelastic attenuation media [7].

The main difference between the DGM and traditional FEM is that the basis function can be discontinuous across elements in DGM. In general, DGM has the following advantages: (i) it maintains good properties with respect to conservation, stability, and convergence; (ii) it is easy to deal with domains with complex structures and non-conforming meshes, allowing for hanging nodes; (iii) the solution can be discontinuous across the element interfaces; (iv) complete localization means that DGM avoids dealing with large global mass matrices and is therefore especially suitable for parallel computing; (v) it is flexible for DGM to deal with locally varying polynomial degrees and element shapes (hp-adaptivity).

Many developed DGMs are based on the semi-discrete approach, wherein they employ the discontinuous Galerkin formulations for the spatial discretization, transforming the original partial differential equations (PDEs) into a system of ordinary differential equations (ODEs). After that, a time-stepping method is carried out to advance the solution in time, such as RK schemes [2], arbitrary high-order derivatives time stepping method [6] or the Lax-Wendroff method [15]. Explicit time-stepping scheme is widely used since it is easy to program. Alternatively, implicit solvers, such as diagonal implicit-RK [16,17] and Newton iterative methods [18,19], are used in time discretization because they permit to use longer time steps. However, the shortcoming of fully implicit solvers is the extremely high computational cost induced by solving the large-scale linear algebraic equations. For this reason, explicit techniques such as the truncated differentiator series method and the predictor-corrector method have been developed (e.g., [20–22]), which turn to be robust methods to convert implicit methods into explicit ones and may preserve the good stability inherent in implicit schemes. In our research these techniques will be considered.