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Higher-Order Rytov Approximation for Large-Scale and Strong Perturbation Media

Bo Feng[1,*], Ru-Shan Wu[2] and Huazhong Wang[1]

[1] *Wave phenomena and inversion imaging group (WPI), School of Ocean and Earth Science, Tongji University, Shanghai, 200092, China.*
[2] *Modeling and Imaging Laboratory, University of California, Santa Cruz, 95060, CA, USA.*

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Abstract. In the field of geophysics, although the first-order Rytov approximation is widely used, the higher-order approximation is seldom discussed. From both theoretical analysis and numerical tests, the accumulated phase error introduced in the first-order Rytov approximation cannot be neglected in the presence of strong velocity perturbation. In this paper, we are focused on improving the phase accuracy of forward scattered wavefield, especially for the large-scale and strong velocity perturbation case. We develop an equivalent source method which can update the imaginary part of the complex phase iteratively, and the higher-order scattered wavefield can be approximated by multiplying the incident wavefield by the exponent of the imaginary part of the complex phase. Although the convergence of the proposed method has not been proved mathematically, numerical examples demonstrate that our method can produce an improved accuracy for traveltime (phase) prediction, even for strong perturbation media. However, due to the neglect of the real part of the complex phase, the amplitude change of the scattered wavefield cannot be recovered. Furthermore, in the presence of multi-arrivals phenomenon, the equivalent scattering source should be handled carefully due to the multi-directions of the wavefield. Further investigations should be done to improve the applicability of the proposed method.

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Key words: Rytov approximation, higher-order approximation, large-scale strong perturbation, forward scattering, equivalent scattering source.

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1 Introduction

From the analysis of scattering characteristics, we know that the forescattering is controlled by the D.C. component (zero-wavenumber part and long wavelength component)

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[*] Corresponding author. *Email addresses:* `ancd111@163.com` (B. Feng), `rwu@ucsc.edu` (R.-S. Wu), `herbhuak@vip.163.com` (H. Wang)

of the medium spectrum [1, 2]. The D.C. component of the medium spectrum increases almost linearly with the propagation distance in general. The validity condition for the Born approximation is the smallness of the scattered field compared with the incident field. Therefore, in the case of long-range propagation of forward scattering, due to the accumulation of phase changes caused by the velocity perturbations, the validity of the Born approximation is easily to be violated. However, phase-change accumulation can be easily handled by the Rytov transformation. This is why the Rytov approximation has been widely used for long distance propagation with only forescattering or small-angle scattering involved, such as the line-of-sight propagation of optical or radio waves [3–5], transmission fluctuations of seismic waves at arrays [6–8], diffraction tomography [9–12], seismic imaging using one-way propagators [13] and the calculation of finite-frequency sensitivity kernel for travel-time tomography [14–17].

It is well-known that for long-range wave propagation in the regime of small-angle scattering, Rytov approximation is superior to the Born approximation. However, the accuracy and convergence of Rytov series is still not established. Some discussions on the validity conditions are in the literatures [4, 18–22]. In the area of diffraction tomography, it is clear that the first-order Rytov approximation is not accurate enough for large strong-contrast media in forward and inverse problems, therefore higher-order Rytov inversion was proposed and the results showed improved inversion accuracy [23]. For statistical wave propagation problem, Manning [24] derived a second-order Rytov approximation for general beam wave propagation through turbulent media. Kim and Tinin [25] used the second-order Rytov approximation to calculate the ionospheric residual error of dual-frequency satellite navigation system, which considered the diffraction arising where the Fresnel radius becomes larger than the inner scale of the spectrum of ionospheric turbulence.

Different from previous researches, we mainly focus on increasing the phase accuracy of the scattered wavefield in order to develop a non-linear traveltime inversion method for strong-perturbation media. This paper is organized as following: In the second section, we will demonstrate how to iteratively approximate the imaginary part of the complex phase function. In the third section, we will use numerical examples to show the improvements of the proposed higher-order Rytov approximation method over the first-order Rytov approximation. Finally the perspectives and limitations of our method are discussed.

2 Method

Let $u_0(\mathbf{x};\omega)$be the solution in the absence of perturbation, i.e.,

$$\left(\nabla^2+k^2\right)u_0=0, \tag{2.1}$$

where the wave-number k is defined as: $k=\omega/c_0(\mathbf{x})$ and $c_0(\mathbf{x})$ is the background velocity, $u_0=u_0(\mathbf{x};\omega)$ is the unperturbed wave field.