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A MULTIDIMENSIONAL FILTER SQP ALGORITHM FOR NONLINEAR PROGRAMMING *

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Abstract

We propose a multidimensional filter SQP algorithm. The multidimensional filter technique proposed by Gould et al. [SIAM J. Optim., 2005] is extended to solve constrained optimization problems. In our proposed algorithm, the constraints are partitioned into several parts, and the entry of our filter consists of these different parts. Not only the criteria for accepting a trial step would be relaxed, but the individual behavior of each part of constraints is considered. One feature is that the undesirable link between the objective function and the constraint violation in the filter acceptance criteria disappears. The other is that feasibility restoration phases are unnecessary because a consistent quadratic programming subproblem is used. We prove that our algorithm is globally convergent to KKT points under the constant positive generators (CPG) condition which is weaker than the well-known Mangasarian-Fromovitz constraint qualification (MFCQ) and the constant positive linear dependence (CPLD). Numerical results are presented to show the efficiency of the algorithm.

Mathematics subject classification: 90C30.
Key words: Trust region, Multidimensional filter, Constant positive generators, Global convergence, Nonlinear programming.

1. Introduction

In this paper, we consider the constrained optimization problem:

$$(\text{NLP}) \quad \begin{cases} \min & f(x) \\ \text{s.t.} & c_{\mathcal{E}}(x) = 0, \\ & c_{\mathcal{I}}(x) \le 0, \end{cases}$$

where $c_{\mathcal{E}}(x) = (c_1(x), c_2(x), \cdots, c_{m_1}(x))^T$, $c_{\mathcal{I}}(x) = (c_{m_1+1}(x), c_{m_1+2}(x), \cdots, c_m(x))^T$, $\mathcal{E} = \{1, 2, \cdots, m_1\}$, $\mathcal{I} = \{m_1+1, m_1+2, \cdots, m\}$, $f : \mathbb{R}^n \to \mathbb{R}$, $c_i : \mathbb{R}^n \to \mathbb{R}$, $i \in \mathcal{E} \cup \mathcal{I}$ are twice continuously differentiable functions.

The sequential quadratic programming (SQP) method has been widely used for solving the problem (NLP) [13] and mathematical programs [24]. It generates a sequence $\{x_k\}$ converging

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to the desired solution by solving the quadratic programming (QP) subproblem:

$$\begin{cases} \min & q(d) = \nabla f(x_k)^T d + \frac{1}{2} d^T B_k d \\ \text{s.t.} & \nabla c_{\mathcal{E}}(x_k)^T d + c_{\mathcal{E}}(x_k) = 0, \\ & \nabla c_{\mathcal{I}}(x_k)^T d + c_{\mathcal{I}}(x_k) \leq 0, \\ & \|d\|_\infty \leq \rho, \end{cases} \tag{1.1}$$

where $\rho > 0$ is the trust region radius, $B_k \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix.

Fletcher and Leyffer [11] proposed a filter SQP method as an alternative to the traditional merit function SQP methods for solving nonlinear programming. The underlying concept is fairly simple. That is the trial step generated from solving a QP subproblem is accepted provided there is a sufficient decrease of the objective function or the constraint violation. In addition, numerical results presented in [11] are very encouraging.

Filter methods have been applied to many current optimization techniques for solving nonlinear programming or even nonlinear semi-definite programming [4,8,15–18,23,26,29,31,32,34,35]. Such methods are characterized by the use of the dominance concept of multi-objective optimization, instead of a penalty parameter whose adjustment can be problematic. Compared with traditional merit function methods, filter methods make search directions accepted more flexible. In the literature, many filter methods for solving constrained optimization, such as [11, 34, 35] etc, are bi-dimensional. A multidimensional filter method was presented by Gould and Toint [18], Gould et al. [14] for nonlinear equations and nonlinear least squares. Subsequently it was applied to solve unconstrained optimization by Gould et al. [17]. In fact, multidimensional filter makes the trial step accepted easily. Some "good steps" rejected by bi-dimensional filter may be accepted by multidimensional filter.

Most of filter methods mentioned above are combined with sequential linear programming (SLP) or SQP type methods. In whatever SLP or SQP type methods, the first-order Taylor expansion is used to linearize the constraints. Traditional SQP merit function methods only combine the objective function with the constraint violation linearly through a penalty parameter. In the process of iteration, only reduction on the merit function is considered while the individual behavior of the constraint violation is ignored. In bi-dimensional filter SQP methods, though the objective function and the constraint violation are considered separately, and a preset upper bound of the constraint violation is used, they consider all the constraints together, which may also ignore individual behaviors of different types of constraints, such as the highly nonlinear (HN) constraints and the nearly linear (NL) ones. Moreover, there exists an undesired link between the objective function and the constraints exists in bi-dimensional methods, which is pointed by Fletcher and Leyffer [12].

We notice that many filter-related algorithms include a feasibility restoration phase for dealing with inconsistency of QP subproblems. Although many methods (e.g. [6, 7, 25]) for solving a nonlinear algebraic system of equalities and inequalities can be used to implement this calculation, it may cost a large amount of computational effort. In addition, global convergence of above methods requires that the Mangasarian-Fromovitz constraint qualification (MFCQ) or the linear independence constraint qualification (LICQ) hold at local solutions of the problem (NLP).

In this paper, inspired by [11, 14], we propose a multidimensional filter method. The constraints are partitioned into p parts and are considered separately. The new filter's entry consists of p parts of constraints. So the conditions for accepting a trial step may be more flexible. An important feature of our algorithm is that the entries in the filter are indepen-