

IMPLICITY LINEAR COLLOCATION METHOD AND ITERATED IMPLICITY LINEAR COLLOCATION METHOD FOR THE NUMERICAL SOLUTION OF HAMMERSTEIN FREDHOLM INTEGRAL EQUATIONS ON 2D IRREGULAR DOMAINS*

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Abstract

In this work, we adapt and compare implicit linear collocation method and iterated implicit linear collocation method for solving nonlinear two dimensional Fredholm integral equations of Hammerstein type using IMQ-RBFs on a non-rectangular domain. IMQs show to be the most promising RBFs for this kind of equations. The proposed methods are mesh-free and they are independent of the geometry of domain. Convergence analysis of the proposed methods together with some benchmark examples are provided which support their reliability and numerical stability.

Mathematics subject classification: 45G10.

Key words: Two dimensional equations, Irregular domain, Fredholm integral equations, Mesh-less method, Numerical treatment.

1. Introduction

In this work, we are interested in using inverse multi-quadric radial basis functions (IMQ-RBFs) for solving Fredholm integral equations of Hammerstein type

$$u(x) = f(x) + \int_{\Omega} K(x, s)\psi(s, u(s))ds, \quad x \in \Omega, \quad (1.1)$$

where Ω denotes a non-rectangular closed subset of R^2 . Throughout the paper, the following assumptions are made:

1. $f \in C(\Omega)$,
2. $K(x, s) \in C(\Omega^2)$,
3. $|\psi^{(0,1)}(s, u_1) - \psi^{(0,1)}(s, u_2)| \leq c_2|u_1 - u_2|$,

where $\psi^{(0,1)}$ is the partial derivative with respect to second variable. Note that the last assumption implies ψ being also Lipschitz with respect to its second variable, i.e.

$$|\psi(s, u_1) - \psi(s, u_2)| \leq c_1|u_1 - u_2|.$$

* Received August 28, 2017 / Revised version received October 4, 2018 / Accepted March 3, 2019 /
Published online June 20, 2019 /

In recent years, mesh-less methods have been widely applied in a number of fields such as multivariate function interpolation and approximation, neural networks and solution of differential and integral equations. The mesh-less methods are based upon the scattered data approximations that estimate a function without any mesh generation on the domain. Among mesh-less methods, the radial basis functions (RBFs) method has become known as a powerful tool for the scattered data interpolation problem. The main advantage of radial basis functions is that they involve a single independent variable regardless of the dimension of the problem [9]. RBFs have been used to approximate the solution of one-dimensional integral equations [5].

In this paper we propose two numerical methods based on inverse multi-quadric radial basis functions (IMQ-RBFs) and Kumar and Sloan method [4] for the solution of nonlinear Fredholm integral equations of Hammerstein type on 2D irregular domains.

The outline of the paper is as follows: In Section 2, we review some basic formulations and properties of RBFs and approximation of a function by these bases. In Sections 3, 4 and 5 we present computational methods for solving (1) utilizing IMQ-RBF approximation together with their error analysis. Finally, numerical experiments are carried out in Section 6.

2. RBF Approximation

A mesh-free method does not require a mesh to discretize the domain of the problem under consideration, and the approximate solution is constructed entirely based on a set of scattered nodes. Radial basis functions is one of the most developed meshless methods that has attracted attention in recent years and form a primary tool for multivariate interpolation [1, 2, 6–8]. It is also receiving increasing attention for solving PDE's on irregular domains. We quote the following definitions from [10].

Definition 2.1. A function $\Phi : \mathbb{R}^d \rightarrow \mathbb{R}$ is said to be radial if there exists a function $\phi : [0, \infty) \rightarrow \mathbb{R}$ such that $\Phi(\mathbf{x}) = \phi(\|\mathbf{x}\|_2)$, for all $\mathbf{x} \in \mathbb{R}^d$.

Some of the most popular RBFs are given in Table 2.1. Suppose $E \subset \mathbb{R}^d$ and let $X =$

Table 2.1: Some well-known functions that generate RBFs.

Name of function	Definition
Gaussian (GA)	$\phi(r) = \exp(-cr^2), \quad c > 0$
Multi-quadrics (MQ)	$\phi(r) = (r^2 + c^2)^{\frac{1}{2}}$
Inverse multi-quadrics (IMQ)	$\phi(r) = (r^2 + c^2)^{\frac{\beta}{2}}, \quad \beta < 0$
Thin plate splines	$\phi(r) = (-1)^{k+1} r^{2k} \log(r), \quad k \in \mathbb{N}$

$\{x_1, \dots, x_N\}$ be a given set of distinct nodal points in E . In interpolation of the scattered data using RBFs, the approximation of a function f usually has the form:

$$\mathcal{P}_N f(x) = \sum_{i=1}^N \lambda_i \phi(\|x - x_i\|), \quad x \in E. \quad (2.1)$$

The interpolation problem is to find $\lambda_i, i = 1, \dots, N$ such that the interpolant $\mathcal{P}_N f$ through all data satisfies

$$\mathcal{P}_N f(x_i) = f(x_i), \quad i = 1, \dots, N. \quad (2.2)$$