

REVIEW ARTICLE

On Current Developments in Partial Differential Equations

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Received 11 February 2020; Accepted 27 February 2020

Abstract. The first part of this article is an overview on some recent major developments in the field of analysis and partial differential equations. It is a brief presentation given by the author at a round table discussion. The second part is a supplement of various details provided by several outstanding researchers on subjects.

AMS subject classifications: 35Q30

Key words: Partial differential equations, harmonic analysis, geometric measure theory.

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1 An overview

During the period of January 5–15, 2020, there was a special program on the analysis and partial differential equations at the Southern University of Science and Technology, China organized by Professor Tao Tang, Xiao-Ming Wang, Linlin Su and myself. In the first seven days, there were special lecture series on the quantitative periodic and stochastic homogenizations and its applications. It is followed by a conference “On the recent trend in the analysis of partial differential equations”. The program also included a rather special Round Table discussion on some important current developments in the field of partial differential equations and the current state of corresponding researches in China.

I am particularly happy that we had such a round table discussions. Many friends and colleagues get together with wonderful conversations about old and new things as well as works in our field of research. Each of us can open its mind to freely express individual views on the past and current developments and visions for the future. Thinking back, it’s truly worth to have such an event once a while. Last year was also special in some sense at my personal level. I arrived at 60, and it’s a natural time to have a reflection. I first returned to China after my Ph.D in USA in 1989, so it is also the 30th year anniversary for me to participate and to organize various forms of academic programs in China. There are a lot great memories and there have been a lot wonderful friends and young collaborators. It has become an important aspect of why I and all of us love mathematics.

On this reflection something has been noticed, for whatever reasons, our generations may have probably not done very well; and history may not necessarily be so friendly. One can no longer have excuses as it had been, at a very different time and situation, for our teacher’s generation 30 years ago. Currently in China, there are thousands of researchers in the field of Analysis & PDE, and in each year over the past decades there have been also in thousands of research papers got published. But one may have to admit that the impacts and visibilities of these works are rather limited. It is not about the activities, but it is not very good. In