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CONVERGENCE RATE OF THE TRUNCATED EULER-MARUYAMA METHOD FOR NEUTRAL STOCHASTIC DIFFERENTIAL DELAY EQUATIONS WITH MARKOVIAN SWITCHING*

Wei Zhang
School of Mathematical Sciences, Heilongjiang University, Harbin, Heilongjiang, China
Email: weizhanghlj@163.com

Abstract

The key aim of this paper is to show the strong convergence of the truncated Euler-Maruyama method for neutral stochastic differential delay equations (NSDDEs) with Markovian switching (MS) without the linear growth condition. We present the truncated Euler-Maruyama method of NSDDEs-MS and consider its moment boundedness under the local Lipschitz condition plus Khasminskii-type condition. We also study its strong convergence rates at time T and over a finite interval $[0, T]$. Some numerical examples are given to illustrate the theoretical results.

Mathematics subject classification: 65L20, 65C40.
Key words: Neutral stochastic differential delay equations, Truncated Euler-Maruyama method, Local Lipschitz condition, Khasminskii-type condition, Markovian switching.

1. Introduction

Due to better explanations of the phenomena, stochastic differential equations (SDEs) and stochastic differential delay equations (SDDEs) which are more efficient and reliable models in dynamical systems receive more and more attention recently. Systems with Markovian switching which were introduced by Kac and Krasovskii (see in [6]) have become a powerful tool in the modelling of practical systems (see in [7, 21] and the references therein). The structures and parameters of the underlying neutral stochastic differential delay equations (NSDDEs) may change abruptly because of environmental disturbances etc. We use the continuous-time Markovian chain $r(t)$ to model such abrupt changes. Thus, the underlying NSDDE

$$d[x(t) - D(x(t-\tau))] = F\left(x(t), x(t-\tau)\right) dt + G\left(x(t), x(t-\tau)\right) dw(t) \tag{1.1}$$

becomes the following NSDDE with Markovian switching (MS), abbreviated as NSDDE-MS

$$d[x(t) - D(x(t-\tau), r(t))] = F\left(x(t), x(t-\tau), r(t)\right) dt + G\left(x(t), x(t-\tau), r(t)\right) dw(t). \tag{1.2}$$

Since most SDEs-MS and SDDEs-MS have no explicit solutions, numerical methods to approximate the underlying solutions become a useful technique. There are many results concerned with the numerical solutions of SDEs-MS and SDDEs-MS in recent years (see e.g., [11, 13, 15, 19, 20, 25–27, 31]). Up to now, most of the strong convergence theories are considered under the global Lipschitz condition or the local Lipschitz condition and the linear

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growth condition. In 2003, Kolmanovskii et al. [8] discussed some important properties of the solutions e.g. boundedness and stability for NSDDEs-MS under Global Lipschitz and linear growth condition. Some numerical solutions to NSDDEs-MS were also discussed in [23, 24, 30]. However, lots of NSDDEs-MS do not satisfy the linear growth condition and there are a few results about NSDDEs-MS without linear growth condition (see, e.g., [9, 10, 22] and the references cited therein). As we know, the efficient explicit Euler schemes of SDEs (even without Markovian switching) can not be convergent in strong sense with the super-linearly growing drift coefficients, see for example [4]. In 2012, Hutzethaler et al. presented tamed Euler method which can solve this problem [5]. In [20], Nguyen et al. proposed tamed-Euler method for hybrid stochastic differential equations with Markovian switching. In 2015 and 2016, Mao [17, 18] presented the truncated Euler-Maruyama (EM) method for SDEs and gave its convergence rate. The truncated (EM) method has been discussed intensively by some authors (see, e.g., [1,3,12]). Recently, we established the strong convergence theory of the partially truncated EM method for a class of SDDEs [28], and considered convergence rates of the truncated EM method for stochastic functional differential equations (SFDEs) in [29] under the local Lipschitz condition plus the Khasminskii-type condition. Though there are many papers on numerical methods for NSDDEs-MS. Up to now, there is little numerical theory on NSDDEs-MS under local Lipschitz condition and Khasminskii-type condition. Obviously, NSDDEs-MS is a generalization of SDEs and SDDEs. However, some new difficulties will appear due to the nonlinear growth condition.

Owning to cheap computational cost, explicit numerical methods are indeed to be discussed. Our aim here is to develop the truncated EM method of NSDDEs-MS under local Lipschitz condition and Khasminskii-type condition. For the convenience, we will, in Section 2, develop the truncated EM method. We will study the moment boundedness in Section 3 and the convergence rate of the truncated EM method in Section 4, respectively. Some numerical examples are given in Section 5. The conclusion of our paper is presented in Section 6.

2. The Truncated Euler-Maruyama Method

Throughout this paper, assume that $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P})$ denotes a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e, it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Let $\mathbb{E}$ be the expectation corresponding to $\mathbb{P}$. Assume that $w(t) = (w_1(t), w_2(t), \cdots, w_m(t))^T$ is an m-dimensional Brownian motion defined on the probability space. Let $|\cdot|$ denote both Euclidean norm in $\mathbb{R}^n$ and the trace norm in $\mathbb{R}^{n\times m}$. Denote by $C([-\tau,0],\mathbb{R}^n)$ the family of continuous functions from $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\tau\leq u\leq 0}|\varphi(u)|$. For $a, b \in \mathbb{R}$, we use $a \vee b$ and $a \wedge b$ for $\max\{a, b\}$ and $\min\{a, b\}$, respectively. If D is a set of Ω, its indicator function is denoted by $\mathbf{1}_D$. $\lfloor x \rfloor$ denotes the biggest integer which is not bigger than x.

Let $r(t)$ $(t \geq 0)$ be a right-continuous Markovian chain on the probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, taking values in a finite state space $S = \{1, 2, \cdots, Q\}$ with $\Gamma = (\gamma_{ij})_{Q\times Q}$ given by

$$\mathbb{P}\{(r(t+\Delta)) = j|r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & if \quad i \neq j, \\ 1 + \gamma_{ij}\Delta + o(\Delta), & if \quad i = j, \end{cases} \tag{2.1}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition from i to j if $i \neq j$, while $\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}$.

Assume that the Markovian chain $r(t)$ is independent of the Brownian motion $w(t)$. It is known that almost every sample path of $r(t)$ is a right-continuous step function with a finite