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Role of Selective Interaction in Wealth Distribution

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Abstract. In our simplified description 'wealth' is money (m). A kinetic theory of a gas like model of money is investigated where two agents interact (trade) selectively and exchange some amount of money between them so that sum of their money is unchanged and thus total money of all the agents remains conserved. The probability distributions of individual money ($P(m)$ vs. m) is seen to be influenced by certain ways of selective interactions. The distributions shift away from Boltzmann-Gibbs like the exponential distribution, and in some cases distributions emerge with power law tails known as Pareto's law ($P(m) \propto m^{-(1+\alpha)}$). The power law is also observed in some other closely related conserved and discrete models. A discussion is provided with numerical support to obtain insight into the emergence of power laws in such models.

Key words: Kinetic theory; selective interaction; disparity; wealth distribution; Pareto's law.

1 Introduction

Econophysics of Wealth distributions is an active area which involves interpreting and analysing real economic data of money, wealth or income distributions of all kinds of people pertaining to different societies and nations [4]. A number of statistical physical models can be found in the literature [1] in connection with the above. Understanding the emergence of Pareto's law ($P(m) \propto m^{-(1+\alpha)}$), now more than a century old, is one of the most important agenda. Some early attempts [5] have been made to understand the wealth distributions, especially the Pareto's law where the index α is generally found to be in the range of 1 to 2.5 more or less universally.

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Some recent works [6,7] assume economic activities to be analogous to elastic collisions to have kinetic theory of gas like models proposed by Chakrabarti and group and later by others (we refer to this class of models as the 'Chakrabarti model'). Analogy is drawn between Money (m) and Energy (E) where temperature (T) is average money ($< m >$) of any individual at 'equilibrium'. There has been a renewed interest in the two-agent exchange model (be it of money, energy or of something else) in the new scenario. For example, a recent work deals with social systems of complex interactions like sex which is based on a granular system of colliding particles (agents) with gaining energy [3].

In this paper we deal with Chakrabarti model kind of systems where it is assumed that any two agents chosen randomly from a total number of agents (N) are allowed to interact (trade) stochastically and thus money is exchanged between them. The interaction is such that one agent wins and the other loses the same amount so that the sum of their money remains constant before and after interaction (trading). Therefore, it is a two-agent zero sum 'game'. This way it ensures the total amount of money of all the agents ($M = \Sigma m_i$) to remain constant. Such a model is thus a conserved model.

2 The models and results

The basic steps of a money exchange (conserved) model are as follows:

$$m_i(t+1) = m_i(t) + \Delta m, \tag{2.1}$$

$$m_j(t+1) = m_j(t) - \Delta m, \tag{2.2}$$

where m_i and m_j are money of the i-th and j-th agents respectively. Here we have t as discrete 'time' which is referred to as a single interaction between two agents. The amount Δm (to be won or to be lost by an agent) is given by the nature of interaction. In a pure gambling, $\Delta m = \epsilon(m_i(t) + m_j(t)) - m_i(t)$, where stochasticity is introduced through the parameter ϵ $(0 < \epsilon < 1)$.

If the agents are allowed to interact for a long enough time an equilibrium distribution of money of individual agents is achieved. We arrive at a Boltzmann-Gibbs type distribution $[P(m) \propto \exp(-m/ < m >)]$ of individual money which is verified numerically. This is quite the same way in which we arrive at the equilibrium energy distribution of a system of gas particles elastically colliding and exchanging energy with each other. The equilibrium temperature corresponds to average money, $< m >$ per agent.

The study is done through numerical computations and the results reported here are obtained with system sizes (=total number of agents) $N = 1000$. In all cases the system is allowed to equilibrate up to $t = 10^6$ time steps. Averaging is done over 1000 realizations in each case. The final distribution of course should not depend on the initial configuration (initial distribution of money among the agents). The wealth distributions we deal with in this paper are ordinary distributions and not cumulative ones. To obtain a distribution we take the average over many different realizations; that is over a number of ways of the random selection of a pair of agents and also over the stochastic term ϵ.