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A Nearly Analytical Discrete Method for Wave-Field Simulations in 2D Porous Media

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Abstract. The nearly analytic discrete method (NADM) is a perturbation method originally proposed by Yang *et al.* (2003) [26] for acoustic and elastic waves in elastic media. This method is based on a truncated Taylor series expansion and interpolation approximations and it can suppress effectively numerical dispersions caused by the discretizating the wave equations when too-coarse grids are used. In the present work, we apply the NADM to simulating acoustic and elastic wave propagations in 2D porous media. Our method enables wave propagation to be simulated in 2D porous isotropic and anisotropic media. Numerical experiments show that the error of the NADM for the porous case is less than those of the conventional finite-difference method (FDM) and the so-called Lax-Wendroff correction (LWC) schemes. The three-component seismic wave fields in the 2D porous isotropic medium are simulated and compared with those obtained by using the LWC method and exact solutions. Several characteristics of wave propagating in porous anisotropic media, computed by the NADM, are also reported in this study. Promising numerical results illustrate that the NADM provides a useful tool for large-scale porous problems and it can suppress effectively numerical dispersions.

Key words: Porous media; nearly-analytic discretization; three-component seismic wavefield; anisotropy; numerical dispersion.

1 Introduction

Usually, the oil/gas reservoir shows fractures, cracks, or pores, so it is a two-phase medium. On the basis of both the two-phase property of porous media and the analyses of the

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solid/fluid interaction and coupling mechanisms, Biot established the theory of the elastic-wave propagation in a porous medium with saturated fluids for the low frequency [1] and the high frequency [2] cases, which is extraordinarily valuable for solving the problem of prospecting for oil and gas. Based on this theory, Biot predicted the slow P-wave that had been verified in the subsequent experiment [19]. To solve Biot's two-phase model and study the wave propagation in porous media with fluids, different numerical methods ([4, 6, 7, 13, 22, 23], others) have been proposed and studied.

Among various methods for seismic modeling in a porous medium, the finite-element method is one of most effective ones [23]. However, it requires expensive computational costs and large storage space compared with the finite-difference method (FDM), which prevents it from solving problems in higher dimension or large models. The FDM for modeling wave propagation is another popular tool due to its rapidity and lesser storage. Unfortunately, the conventional finite-difference methods with orders 2 and 4 often suffer from seriously numerical dispersion when too few samples per wavelength are used or when the models have large velocity contrast, or artifacts caused by source at grid points [9, 25]. Higher order FD (finite difference) methods such as high-order compact FD schemes also have numerical dispersions, and generally involve much more grid points when computing a displacement value at a grid-point (e.g. [5, 21]). Although numerical dispersions can also be suppressed via using a flux-corrected transport (FCT) technique [27], the FCT method can hardly recover the lost resolution by the numerical dispersion when the spatial sampling becomes too coarse [25]. On the other hand, acoustic and elastic waves have inherent dispersions as the waves propagate in a porous medium with fluids. This implies that two kinds of dispersions (numerical dispersion and wave dispersion) might appear simultaneously in wave fields if the conventional FD methods are used to compute the wave fields in a porous medium. In such a case, it is not a good idea to use the FCT technique to eliminate the numerical dispersions because we do not know how to choose the proper control parameters used in the FCT method for suppressing the numerical dispersions [25, 30]. The pseudo-spectral method [12, 15, 16] is attractive as the space operators are exact up to the Nyquist frequency. However, it requires the Fourier transform of the wave-field, which is computationally expensive for 3-D models and has difficulty in handing sharp boundaries [18]. The so-called nearly analytical discrete method (NADM), recently developed for solving acoustic and elastic equations [24, 26] and initially reported by Konddoh *et al.* (1994) [14] for solutions of parabolic and hyperbolic equations, can effectively eliminate the numerical dispersions without any additional treatments and has very high numerical accuracy. Moreover, compared with the conventional FD and finite-element methods for the single-phase case, the NADM requires less memory and is computationally cheaper [26].

Based on the above-mentioned points, in this paper we try to extend the NADM for the single-phase case to the two-phase case and present the robust NADM that is suitable for simulating acoustic and elastic waves propagating in 2D fluid-saturated porous media. Although the extension is not a difficult task, it is still far from being straightforward. To derive the NADM for the underlying problem, we first divide the order-two time derivatives