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Potts Model with q States on Directed Barabási-Albert Networks

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Abstract. On directed Barabási-Albert networks with two and seven neighbours selected by each added site, the Ising model with spin $S=1/2$ was seen not to show a spontaneous magnetisation. Instead, the decay time for flipping of the magnetisation followed an Arrhenius law for Metropolis and Glauber algorithms, but for Wolff cluster flipping the magnetisation decayed exponentially with time. However, on these networks the Ising model spin $S=1$ was seen to show a spontaneous magnetisation. In this case, a first-order phase transition for values of connectivity $z=2$ and $z=7$ is well defined. On these same networks the Potts model with $q=3$ and 8 states is now studied through Monte Carlo simulations. We also obtained for $q=3$ and 8 states a first-order phase transition for values of connectivity $z=2$ and $z=7$ for the directed Barabási-Albert network. Theses results are different from the results obtained for the same model on two-dimensional lattices, where for $q=3$ the phase transition is of second order, while for $q=8$ the phase transition is of first-order.

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Key words: Monte Carlo simulation, Ising, networks, disorder.

1 Introduction

Sumour and Shabat [1,2] investigated the Ising models with spin S=1/2 on the directed Barabási-Albert networks [3] with the usual Glauber dynamics. No spontaneous magnetisation was found, in contrast to the case of undirected Barabási-Albert networks [4–6] where a spontaneous magnetisation was found lower a critical temperature which increases logarithmically with system size. In S=1/2 systems on undirected, scale-free hierarchical-lattice small-world networks [7], conventional and algebraic (Berezinskii-Kosterlitz-Thouless) ordering, with finite transition temperatures, have been found. Lima

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and Stauffer [8] simulated directed square, cubic and hypercubic lattices in two to five dimensions with heat bath dynamics in order to separate the network effects form the effects of directedness. They also compared different spin flip algorithms, including cluster flips [9], for Ising-Barabási-Albert networks. They found a freezing-in of the magnetisation similar to [1,2], following an Arrhenius law at least in low dimensions. This lack of a spontaneous magnetisation (in the usual sense) is consistent with the fact that if on a directed lattice a spin S_j influences spin S_i, then spin S_i in turn does not influence S_j, and there may be no well-defined total energy. Thus, they show that for the same scale-free networks, different algorithms give different results. More recently, Lima [10,11] simulated the Ising model for spin $S=1$ on the directed Barabási-Albert network and different from the Ising model for spin $S=1/2$, the order-disorder phase transition of order parameter is well defined in this system. He obtained a first-order phase transition for values of connectivity $z=2$ and $z=7$ of the directed Barabási-Albert network. Now we study the Potts model for $q=3$ and 8 on the directed Barabási-Albert network for values of connectivity $z=2$ and $z=7$. Unlike the Ising model for spin $S=1/2$, the order-disorder phase transition of order parameter is well defined in this system. We obtained a first-order phase transition for values of connectivity $z=2$ and $z=7$ for the directed Barabási-Albert network.

2 Model and simulation

We consider the Potts model with $q=3$ and 8 states, on the directed Barabási-Albert Networks, defined by a set of spin variables σ taking the values 1, 2 and 3 for $q=3$, and $\sigma=1,\cdots,8$ for $q=8$ situated on every site of a directed Barabási-Albert Network with N sites.

The Potts interaction energy is given by

$$E=-J\sum_i\sum_k\delta_{\sigma_i\sigma_k} \tag{2.1}$$

where k-sum runs over all nearest neighbors of i. In this network, each new site added to the network selects z already existing sites as neighbours influencing it; the newly added spin does not influence these neighbours. To study the critical behavior of the model we define the variable $e=E/N$ and $m=(q\cdot\max_i[n_i]-N)/(q-1)$, where $n_i\leq N$ denotes the number of spins with "orientation" $i=1$, 2 and 3 for $q=3$ and $i=1,2,\cdots,8$ for $q=8$, in one network configuration. From the variable energy we can compute the average energy, the specific heat and the energetic fourth-order cumulant,

$$u(K)=[<E>]_{av}/N, \tag{2.2}$$

$$C(K)=K^2N[<e^2>-<e>^2]_{av}, \tag{2.3}$$

$$B_i(K)=\left[1-\frac{<e^4>}{3<e^2>^2}\right]_{av}, \tag{2.4}$$