COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **2**, No. 4, pp. 577-610

Commun. Comput. Phys.
August 2007

Review Article

The Unified Coordinate System in Computational Fluid Dynamics

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Received 17 May 2006; Accepted (in revised version) 10 November 2006

Communicated by Chi-Wang Shu

Available online 8 January 2007

Abstract. A fundamental issue in CFD is the role of coordinates and, in particular, the search for "optimal" coordinates. This paper reviews and generalizes the recently developed unified coordinate system (UC). For one-dimensional flow, UC uses a material coordinate and thus coincides with Lagrangian system. For two-dimensional flow it uses a material coordinate, with the other coordinate determined so as to preserve mesh othorgonality (or the Jacobian), whereas for three-dimensional flow it uses two material coordinates, with the third one determined so as to preserve mesh skewness (or the Jacobian). The unified coordinate system combines the advantages of both Eulerian and the Lagrangian system and beyond. Specifically, the followings are shown in this paper. (a) For 1-D flow, Lagrangian system plus shock-adaptive Godunov scheme is superior to Eulerian system. (b) The governing equations in any moving multi-dimensional coordinates can be written as a system of closed conservation partial differential equations (PDE) by appending the time evolution equations – called geometric conservation laws – of the coefficients of the transformation (from Cartesian to the moving coordinates) to the physical conservation laws; consequently, effects of coordinate movement on the flow are fully accounted for. (c) The system of Lagrangian gas dynamics equations is written in conservation PDE form, thus providing a foundation for developing Lagrangian schemes as moving mesh schemes. (d) The Lagrangian system of gas dynamics equations in two- and three-dimension are shown to be only weakly hyperbolic, in direct contrast to the Eulerian system which is fully hyperbolic; hence the two systems are not equivalent to each other. (e) The unified coordinate system possesses the advantages of the Lagrangian system in that contact discontinuities (including material interfaces and free surfaces) are resolved sharply. (f) In using the UC, there is no need to generate a body-fitted mesh prior to computing flow past a body; the mesh is automatically generated by the flow. Numerical examples are given to confirm these properties. Relations of the UC approach with the Arbitrary-Lagrangian-Eulerian (ALE) approach and with various moving coordinates approaches are also clarified.

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PACS (2006): 47.11.-j, 47.40.-x

Key words: Unified coordinates, Eulerian coordinates, Lagrangian coordinates, contact discontinuities, automatic mesh generation, moving mesh, conservation form.

Contents

1 Introduction

Computational Fluid Dynamics (CFD) uses large scale numerical computation to solve problems of fluid flow. It has been known since its onset that the numerical solution to a given flow depends on the relation between the flow and the coordinates (mesh) used to compute it. Each of the two well-known coordinate systems for describing fluid flow – Eulerian and Lagrangian – has advantages as well as drawbacks. Eulerian method is relatively simple, but its drawbacks are: (a) it smears contact discontinuities badly, and (b) it needs generating a body-fitted mesh prior to computing flow past a body. Lagrangian method, by contrast, resolves contact discontinuities (including material interfaces and free surfaces) sharply, but it too has drawbacks: (a) the gas dynamics equations could not be written in conservation partial differential equations (PDE) form, rendering numerical computation complicated, and (b) it breaks down due to cell deformation.

The objective of this paper is to review and generalize the recently developed unified coordinate system (UC) mostly by the author and his collaborators [1–16]. To put it in perspective we shall first comment on the relative merits of the existing coordinate systems, mainly, Eulerian, Lagrangian, Arbitrary-Lagrangian-Eulerian (ALE), and the moving mesh (coordinate).

1.1 Theoretical issues

For more than 200 years, two coordinate systems have existed for describing fluid flow: Eulerian system is fixed in space, whereas Lagrangian system follows the fluid. An immediate question is "are they equivalent to each other theoretically?" This question must