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A Numerical Comparison of Outflow Boundary Conditions for Spectral Element Simulations of Incompressible Flows

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Abstract. Outflow boundary conditions (OBCs) are investigated for calculation of incompressible flows by spectral element methods. Several OBCs, including essential-type, natural-type, periodic-type and advection-type, are compared by carrying out a series of numerical experiments. Especially, a simplified form of the so-called Orlanski's OBCs is proposed in the context of spectral element methods, for which a new treatment technique is used. The purpose of this paper is to find stable low-reflective OBCs, suitable and flexible for use of spectral element methods in simulation of incompressible flows in complex geometries. The computation is firstly carried out for a 2D simulation of Poiseuille-Bénard channel flow with Re=10, Ri=150 and Pr=2/3. This flow serves as a useful example to demonstrate the applicability of the proposed OBCs because it exhibits a feature of vortex shedding propagating through the outflow boundary. Then a 3D flow around an obstacle is computed to show the efficiency in the case of more general geometries. Among the tested OBCs, the advection-type OBCs are proven to have better behavior as compared with the others.

AMS subject classifications: 65M70, 74S25, 76M22

Key words: Spectral element methods, outflow boundary conditions, incompressible flows.

1 Introduction

For the numerical solution of channel or external viscous flow problems, we often have to truncate the original unbounded domain into a bounded one in order to make the problems computable. By doing that, we need to introduce appropriate artificial boundary conditions (ABCs) on the artificial external boundary for the closure purpose. The

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ABCs must meet a fundamental requirement: a unique solution inside the finite computational domain exists and can be computed within the desired accuracy as compared to the original solution given by the infinite domain problem.

This paper is focused on outflow ABCs (OBCs), which are critical in many flow simulations. It has been observed that unsuitable OBCs may result in undesired artificial boundary layer, which furthermore render the calculation unstable. On the other hand, the construction of the exact OBCs is generally very difficult. There exists much work concerning numerical and theoretical investigations on the OBCs, mainly done in the framework of finite difference and finite element methods, see for example [2, 11, 16, 25, 29, 30, 34]. In [26], five OBCs are compared in the framework of finite volume methods. Roughly speaking, these OBCs can be classified into three categories: essential-type, natural-type and Orlanski's type [27]. Similar to many other investigations [1, 17, 27], the numerical experiments presented in [26] show that the OBC of the Orlanski's type is a low-reflective boundary condition. We refer to [1, 36] for a review of OBCs for elliptic flows.

The aim of this paper is to compare several OBCs for the computation of certain unsteady channel or external flows by using spectral element methods. The tested OBCs include the traditional boundary condition of essential-type and natural-type, as well as of advection-type (a simplified form of the so-called Orlanski's boundary condition). The use of the latter is based on an essential assumption saying that the Navier-Stokes equations can be linearized in the far field against the free-stream background. We will check the behavior of the advection-type OBCs by discussing its implementation method and accuracy in the context of spectral element methods. It is known that the choice of OBCs is related to the numerical method used in the simulation. Different methods treat the boundary conditions in different manners. In other words, the boundary conditions suitable for the finite difference method are not necessarily applicable to the finite element or spectral methods. Generally, low-order methods, like finite difference methods, treat the boundary conditions in a more arbitrary way as compared to the high order methods, like spectral methods. The latter is heavily based on the variational formulation, and requires global integration of the boundary conditions into the uniform formulation. It is well-known that when the solution of the problem is sufficiently smooth, the convergence of the spectral method is exponential. However, to keep this convergence rate, the treatment of the boundary conditions must be as accurate as the spectral approximation.

In this paper, we restrict ourselves to the following OBCs: 1) periodic conditions; 2) essential OBCs; 3) natural OBCs, $\frac{\partial \varphi}{\partial n}=0$; 4) advection OBCs, $\frac{\partial \varphi}{\partial t}+V\frac{\partial \varphi}{\partial n}=0$. In the above expressions, φ stands for the related variables (mostly, the velocities or temperature), V is a vector to be determined, n is the outward normal on the outflow boundary (hereafter we use letters of boldface type to denote vectors and vector functions). Among these OBCs, only the essential one is a standard boundary condition for spectral element approximations to the Navier-Stokes equations; the others have to be modified or transformed before application. Especially, we will propose an implementation technique to deal with the advection OBCs in the framework of spectral element methods. For com-