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Fully Kinetic, Electromagnetic Particle-in-Cell Simulations of Plasma Microturbulence

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Abstract. A novel numerical method, based on physical intuition, for particle-in-cell simulations of electromagnetic plasma microturbulence with fully kinetic ion and electron dynamics is presented. The method is based on the observation that, for low-frequency modes of interest [$\omega/\omega_{\rm ci} \ll 1$, ω is the typical mode frequency and ω_{ci} is the ion cyclotron frequency] the impact of particles that have velocities larger than the resonant velocity, $v_r \sim \omega/k_{\|}$ ($k_{\|}$ is the typical parallel wavenumber) is negligibly small (this is especially true for the electrons). Therefore it is natural to analytically segregate the electron response into an adiabatic response and a nonadiabatic response and to numerically resolve only the latter: this approach is termed the splitting scheme. However, the exact separation between adiabatic and nonadiabatic responses implies that a set of coupled, nonlinear elliptic equations has to be solved; in this paper an iterative technique based on the multigrid method is used to resolve the apparent numerical difficulty. It is shown that the splitting scheme allows for clean, noise-free simulations of electromagnetic drift waves and ion temperature gradient (ITG) modes. It is also shown that the advantage of noise-free kinetic simulations translates into better energy conservation properties.

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Key words: Plasma micro-turbulence, particle-in-cell simulation, multigrid solver.

1 Introduction

There is growing experimental [2, 3] and theoretical [4, 6] evidence that the so-called anomalous (cross-field) transport observed in toroidal fusion devices is due to microturbulence (for a good review on the topic of anomalous transport, the reader should consult

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the paper by Horton [5] and references therein). Although there is now a good understanding of the basic mechanisms of small-scale, low-frequency turbulence in tokamak and stellarator plasmas [7], the actual numerical modeling of such turbulent plasmas is lagging behind the theory. In particular most numerical studies rely on the assumption that the electrons respond adiabatically to the waves; such an assumption is of course very useful but also implies not addressing some key physical effects associated with kinetic (non-adiabatic) electrons.

In order to take into account wave-particle interactions and nonlinear wave effects [8] we adopt the particle-in-cell (PIC) simulations approach to simulate the ion dynamics *and* the electron dynamics. As it is well known, in view of the large mass ratio, $m_i/m_e \gg 1$, the PIC simulation of electron dynamics suggests the use of a very small time step of integration. However our main interest is to simulate low-frequency ($\omega/\omega_{ci} \ll 1$), drift-type ($k_{||}/k_{\perp} \ll 1$) modes for which the bulk of the electrons respond adiabatically to the waves [4,5]. Therefore it may prove advantageous to focus on the *nonadiabatic* part of the electron response rather than on the entire electron response: this is the basic idea of the splitting scheme. In the electrostatic case the splitting scheme has been shown to be more accurate in the linear regime [26] (e.g. linear growth rates) and in the nonlinear regime [1] (e.g. energy conservation in the saturated state) than the conventional δf scheme [10,11].

This paper presents a generalization of the splitting scheme to the electromagnetic case. The electromagnetic splitting scheme is a natural extension of the electrostatic version of the scheme [26]; however there are new numerical difficulties that require special consideration. In addition, we show how to account for collisional effects (which were neglected in the electrostatic case [1,26]) in the splitting scheme.

The paper is organized as follows. In Section 2, the derivation of the electromagnetic splitting scheme is presented and, for the sake of comparison, the model equations for the conventional δf method are also given. The equations governing the required scalar fields (e.g. electrostatic potential) are also presented in the same section. Section 3 is devoted to linear benchmarks. A potential numerical instability is discussed in detail in Section 4. Nonlinear simulations of electromagnetic drift waves using the splitting scheme are presented in Section 5. Concluding remarks are given in Section 6.

2 Splitting scheme for electromagnetic turbulence

2.1 Basic method

The distribution function for particle species j, denoted F_j, is governed by the collisional, gyrokinetic Vlasov equation (in the long-wavelength limit)

$$\frac{dF_j}{dt} \equiv \frac{\partial F_j}{\partial t} + \left(v_{\|}\widehat{\mathbf{b}} + \mathbf{V}_E\right) \cdot \boldsymbol{\nabla} F_j + \frac{q_j}{m_j} E_{\|} \frac{\partial F_j}{\partial v_{\|}} = C_j(F_j), \tag{2.1}$$