

Multiscale Finite Element Modelling of Flow Through Porous Media with Curved and Contracting Boundaries to Evaluate Different Types of Bubble Functions

V. Nassehi^{1,*}, M. Parvazinia² and A. Khan¹

¹ *Department of Chemical Engineering, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK.*

² *Department of Mathematical Modelling and Simulation, Iran Polymer and Petrochemical Institute, Tehran, P.O. Box 14185/458, Iran.*

Received 15 August 2006; Accepted (in revised version) 9 November 2006

Communicated by Pingwen Zhang

Available online 22 January 2007

Abstract. The Brinkman equation is used to model the isothermal flow of the Newtonian fluids through highly permeable porous media. Due to the multiscale behaviour of this flow regime the standard Galerkin finite element schemes for the Brinkman equation require excessive mesh refinement at least in the vicinity of domain walls to yield stable and accurate results. To avoid this, a multiscale finite element method is developed using bubble functions. It is shown that by using bubble enriched shape functions the standard Galerkin method can generate stable solutions without excessive near wall mesh refinements. In this paper the performances of different types of bubble functions are evaluated. These functions are used in conjunction with bilinear Lagrangian elements to solve the Brinkman equation via a penalty finite element scheme.

PACS (2006): 47.11.-j, 47.11.Fg, 47.11.St

Key words: Finite element, multiscale method, porous media, Newtonian fluid flow, bubble function, static condensation.

1 Introduction

In a porous medium, flow can be represented by different types of governing equations depending on the range of the permeability of the domain and the flow Reynolds number [1]. In highly permeable porous media, low Reynolds number flow regimes can be

*Corresponding author. *Email addresses:* V.Nassehi@lboro.ac.uk (V. Nassehi), A.Khan@lboro.ac.uk (M. Parvazinia), m.parvazinia@ippi.ac.ir (A. Khan)

represented by the Brinkman equation. In this type of flow where permeability of porous matrix is high the fluid carries some of the imposed stress. This effect rises sharply in near wall layers as the permeability of porous media decreases. It is interpreted as the flow system having different scales, a "fine scale" in the near wall zone and a "coarse scale" in the rest of the domain. Therefore, theoretically accurate modelling of the Brinkman regime can only be obtained via excessive mesh refinement of the solution domain, at least in the region of the boundary layer [2]. However, the thickness of the boundary layer is not known a priori and depends on the domain permeability. This in turn makes the classical schemes such as the standard Galerkin method unsuitable for a multiscale problem such as the Brinkman equation [3]. These types of problems can be modelled using multiscale variational methods [4, 5]. These techniques are currently used to solve problems related to turbulent flows, structural analysis of composite materials, flow through porous media, weather forecasting and large-scale molecular dynamic simulations. Representation of all physical scales need a high level of discretisation which is a common difficulty with these problems. To have stable-accurate solution, the multiscale method should be capable of incorporating the influence of the fine-scales while using discretisation at a coarse level to avoid excessive mesh refinement. In a two-scale method, the field unknown is divided into two parts as $u = u_1 + u_b$, where u_b is known as fine, sub-grid or unresolved scale which may be derived analytically and u_1 is known as coarse or resolved scale where is represented by a standard polynomial finite element approximation.

In spite of theoretical progresses in this area the development of algorithms which enable implementation of the theoretical considerations in practice is not a trivial matter [6]. Bubble functions can be incorporated in a finite element discretisation to generate a multiscale scheme. These functions are, typically, high order polynomials which vanish on the element boundaries [7–11]. The bubble functions can be systematically derived using the residual free bubble method [12–15]. The essential idea of this method is that the bubble functions should satisfy, strongly, the model differential equation within each element subject to homogeneous boundary conditions. In multi-dimensional problems, the analytical solution of a partial differential equation (PDE) in the residual free method within each element is a major task. The analytical solution of a PDE can be replaced by the analytical solution of an ODE (ordinary differential equation), in the residual free bubble function method. To this end the exact solution of the ODE is approximated by the Taylor series expansion and the multi-dimensional bubble functions are derived by tensor product of one dimensional bubbles.

In the present work, a continuous penalty scheme is used to evaluate polynomial bubble functions in multiscale finite element solution of the flow in porous media with curved and contracting boundaries using the Brinkman equation. The method of incorporating bubble functions with Lagrangian shape functions using static condensation method, derivation of two dimensional bubble functions and elimination of the boundary integrals are explained. The numerical results are validated with analytical solution in a simple rectangular domain and then the isothermal flow of a Newtonian fluid is studied in different domains.