

Analysis and Numerical Solution of Transient Electromagnetic Scattering from Overfilled Cavities

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Received 12 December 2005; Accepted (in revised version) 8 May 2006

Abstract. A hybrid finite element (FEM) and Fourier transform method is implemented to analyze the time domain scattering of a plane wave incident on a 2-D overfilled cavity embedded in the infinite ground plane. The algorithm first removes the time variable by Fourier transform, through which a frequency domain problem is obtained. An artificial boundary condition is then introduced on a hemisphere enclosing the cavity that couples the fields from the infinite exterior domain to those inside. The exterior problem is solved analytically via Fourier series solutions, while the interior region is solved using finite element method. In the end, the image functions in frequency domain are numerically inverted into the time domain. The perfect link over the artificial boundary between the FEM approximation in the interior and analytical solution in the exterior indicates the reliability of the method. A convergence analysis is also performed.

Key words: Time domain; overfilled cavity; scattering; Fourier transform; finite element method.

1 Introduction

Transient Maxwell's equations for electromagnetic scattering problems have been studied extensively both in theory and computation. One of the important advantages of these equations over their time-harmonic counterparts is that one can obtain the scattering properties such as radar cross sections (RCS) of scatterers for a wide range of frequencies with a single analysis by an application of Fourier transform. Integral equation methods with retarded potentials have been analyzed and successfully implemented for both perfectly conducting and homogeneous dielectric bodies (See, for example, [1] and references

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therein). For the more complex heterogeneous scattering bodies, variational methods are easier to be formulated and simulated. Among the scattering problems, of particular interest is the study of electromagnetic scattering from cavities and the calculation of their RCS. This is because cavity RCS often dominates a target's overall RCS and is computationally challenging. One of the main difficulties in numerically approximating solutions involving cavities is the appearance of spurious modes caused by interior resonances. A variety of techniques have been developed to simulate the scattering by cavities. They include high and low frequency methods [5, 8, 12], the method of moments [19, 21], the time domain finite difference method [6], and hybrid methods [9–11, 13, 14]. Mathematical treatment of scattering problems involving cavities can be found in [2–4, 16]. It's a common assumption that the cavity opening coincides with the aperture on an infinite ground plane, and hence simplifying the modelling of the exterior (to the cavity) domain. This severely limits the application of these methods since many cavity openings are not planar. In [18], a mathematical model characterizing the scattering by over-filled cavities was developed and proved well-posed. In particular, the method decomposes the entire infinite solution domain to two sub-domains: the infinite upper half plane over the perfect electrically conducting (PEC) ground plane exterior to the hemisphere enclosing the cavity aperture, and the cavity plus the hemisphere region. The problem is solved exactly in the infinite sub-domain, while the other is solved using finite elements. The two regions are coupled over the hemisphere via the introduction of a boundary operator exploiting the field continuity over material interfaces.

Other methods designed to numerically determine the fields scattered by obstacles/cavities which give rise to infinite computational domains include absorbing boundary conditions (ABC), perfectly matched layer (PML), and transparent boundary conditions (for a survey of non-reflecting boundary conditions see [7]). In [15], transient Maxwell's equations for the electric field are first discretized in time by Newmark's time-stepping scheme. Then, at each time step a nonlocal boundary condition is imposed on the cavity opening, i.e. the interface between the cavity and the upper half space, to enable the scattering problem to be restricted to the cavity itself. A variational formulation of the problem is derived and shown to possess a unique weak solution at each time step. In [17], the variational problem is fully discretized by first-order edge elements of the first type. There, approximating problems are shown to admit unique solutions and they converge to the exact solutions provided the cavity is characterized by a regular electric permittivity ε .

In this paper, we first apply Fourier transform with respect to time t to the governing wave equation, resulting in a Helmholtz equation in the frequency domain. We then analytically solve the exterior problem via Fourier transform, getting an analytical formula which is incorporated in the boundary condition of the variational problem defined in the interior domain. The interior problem is subsequently solved via the finite element method. At last, the image functions in the frequency domain are numerically inverted into the real time domain by employing the cubic spline interpolation. We also investigate the convergence property of the method.