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Approximate Boundary Conditions for Patch Antennas Mounted on Thin Dielectric Layers

Habib Ammari[1,*], Hyeonbae Kang[2] and Eunjoo Kim[1]

[1] *Centre de Mathématiques Appliquées, CNRS UMR 7641 and Ecole Polytechnique, 91128 Palaiseau Cedex, France.*
[2] *Department of Mathematical Sciences and RIM, Seoul National University, Seoul 151-747, Korea.*

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Abstract. In this paper we discuss scattering problems inherent in curved microstrip structures mounted on thin dielectric structures. We provide approximate boundary conditions for such structures in the framework of integral equations.

Key words: Approximate boundary conditions; boundary integral method; thin dielectric layer; patch antenna.

1 Introduction

The solution of the scattering problem of a plane wave by a metallic target coated by a thin dielectric substrate requires to solve a system of integral equations coupling the solution in the gain and the surrounding medium. Then, the resulting method is both time and memory consuming. Moreover, since the scale of the spatial change in electromagnetic fields in the direction of the thickness of such a thin layer is considerably different from that in the transverse directions, such a direct formulation suffers from numerical instabilities if the dielectric layer is too thin. An alternative to this approach consists in approximately simulating the interior propagation phenomenon by the way of a boundary conditions, the so-called approximate boundary conditions, set on the surface of the obstacle and next to solve the associated scattering problem. See [3, 4, 10, 12].

In this paper we discuss scattering problems inherent in curved microstrip structures mounted on thin dielectric layers. These structures are widely used in printed-circuit technology, microwave integrated circuits, and the antenna industry [7, 9, 11, 14]. It has

*Correspondence to: Habib Ammari, Centre de Mathématiques Appliquées, CNRS UMR 7641 and Ecole Polytechnique, 91128 Palaiseau Cedex, France. Email: ammari@cmapx.polytechnique.fr

been difficult to analyze electromagnetic fields around such structures. Indeed, the classical approximate boundary conditions do not provide an accurate approximation of the electromagnetic fields due to the fact that the presence of the microstrip patch causes a change in the relation between the electromagnetic fields at the dielectric interface.

This paper extends the concept of approximate boundary conditions to microstrip structures and gives a detailed mathematical derivation of an approximate boundary condition for a microstrip patch in the framework of integral equations.

2 Problem formulation

Let Ω be a bounded domain in $\mathbb{R}^2$, with a connected $\mathcal{C}^{2,\alpha}, 0 < \alpha < 1$, boundary. For $h > 0$, let $\Omega_d := \{x \in \mathbb{R}^2 \setminus \overline{\Omega} : \text{dist}(x, \partial\Omega) < h\}$ and $\Omega_e := \mathbb{R}^2 \setminus \overline{(\Omega \cup \Omega_d)}$. We assume that the outer part of $\partial\Omega_d$ consists of two disjoint parts Γ and Γ_e so that $\partial\Omega_d = \Gamma \cup \Gamma_e \cup \partial\Omega$. Put $\Gamma_0 := \{x \in \partial\Omega : x + h\nu_x \in \Gamma\}$, where ν denotes the outward normal to $\partial\Omega$. The domain Ω_d represents the thin dielectric structure while Γ represents the antenna patch mounted on it.

The profiles of electric permeability and permittivity are given by

$$\mu_h(x) = \begin{cases} \mu_d, & x \in \Omega_d, \\ \mu_e, & x \in \Omega_e, \end{cases}$$

and

$$\epsilon_h(x) = \begin{cases} \epsilon_d, & x \in \Omega_d, \\ \epsilon_e, & x \in \Omega_e, \end{cases}$$

respectively, where μ_d, μ_e, ϵ_d and ϵ_e are positive constants. If we allow the degenerate case $h = 0$, then the functions $\mu_h(x)$ and $\epsilon_h(x)$ are equal to the constants μ_e and ϵ_e.

Let $k_d := \omega\sqrt{\mu_d \epsilon_d}$ and define k_e likewise. For a given incident wave E_i, let E_h^Γ denote the solution to the scattering problem

$$\nabla \cdot \frac{1}{\mu_h} \nabla E_h^\Gamma(x) + \omega^2 \epsilon_h E_h^\Gamma(x) = 0, \quad x \in \Omega_e \cup \Omega_d,$$

with the radiation condition

$$\lim_{|x|\to\infty} \sqrt{|x|} \left(\frac{\partial (E_h^\Gamma - E_i)(x)}{\partial |x|} - i\omega k_e (E_h^\Gamma - E_i)(x) \right) = 0,$$

and the Dirichlet boundary condition

$$E_h^\Gamma = 0 \quad \text{on } \partial\Omega \cup \Gamma. \tag{2.1}$$