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Some Techniques for Computing Wave Propagation in Optical Waveguides

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Abstract. Optical wave-guiding structures that are non-uniform in the propagation direction are fundamental building blocks of integrated optical circuits. Numerical simulation of lightwaves propagating in these structures is an essential tool to engineers designing photonic components. In this paper, we review recent developments in the most widely used simulation methods for frequency domain propagation problems.

Key words: Optical waveguides; operator marching method; mode matching method; beam propagation method; bidirectional propagation methods; rational approximations.

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1 Introduction

Optical waveguides [1–3] are structures that guide the propagation of light. They are fundamental building blocks of optical communications systems [4] and integrated optical circuits [5]. For a straight waveguide which is invariant along the waveguide axis (denoted by z in this paper), the basic issue is to analyze the mode structures at a fixed frequency. A propagating mode of a straight waveguide is a special solution of the Maxwell's equations that depends on z as $e^{i\beta z}$ and decays to zero as the transverse variables (x and y) tend to infinity. For a lossless medium, the propagation constant β is real. The problem of computing the modes is an eigenvalue problem where β^2 is the eigenvalue. An optical waveguide is typically an open structure, that is, its cross section is the entire xy-plane. As a result, a general wave field in a straight waveguide contains not only the propagating modes, but also a continuum (represented as an integral) of the radiation and the

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evanescent modes. A general three-dimensional optical waveguide may also have complex modes [6, 7]. The propagation constants of these modes are complex and their mode profiles decay to zero at infinity. Analytic solutions of waveguides modes are only available in a few simple cases. Numerical methods [8–10] are needed for computing the modes of most practical waveguides.

Optical waveguides (or general wave-guiding structures) that are non-uniform in z are important for integrated optics [5]. For example, a bent waveguide is used to turn the propagation direction, an S-bend is used to introduce a lateral displacement, a Y-branch is used to split one waveguide into two, a taper is needed to connect two waveguides of different sizes, waveguide gratings are introduced for various purposes such as filters and reflectors. To simulate the lightwave propagation in these z-varying wave-guiding structures, accurate and efficient numerical or analytic methods are needed. The problem is more difficult since the z-variable is no longer separated, except when the structure is a bent waveguide with a constant bending curvature. In that case, the variable z can be defined along the bend and be separated again. For a general z-varying wave-guiding structure, the frequency domain propagation problem is a boundary value problem. Assuming that the structure is z-invariant for $z < 0$ and $z > a$, we can impose boundary conditions at $z = 0$ and $z = a$. The length of the structure a is usually much larger than the typical wavelength. In some cases, a may be a few millimeters, but the free space wavelength λ_0 is on the order of a micrometer. Since a certain number of grid points (or basis functions) are needed for each wavelength, standard numerical methods that discretize the whole wave-guiding structure are prohibitively expensive.

Fortunately, a number of special features are available for typical optical waveguides. Some efficient numerical and analytical methods have been developed to take advantage of these features. Firstly, although the cross section of an open optical waveguide is the entire xy-plane, the size of the waveguide core is on the order of λ_0 and it is much smaller than a. Using the powerful perfectly matched layer (PML) [11, 12] technique, the transverse plane can be truncated to a relatively small region. Therefore, the propagation problem is formulated in a domain with just one direction (i.e. z) having a particularly large length. This special geometric feature gives rise to *marching methods* that reformulate (exactly or approximately) the original boundary value problem as initial value problems in z. Exact reformulations are developed for pairs of operators and they will be referred to as the operator marching methods (OMM). Secondly, many structures such as waveguide tapers, bent waveguides, S-bends and even Y-branches change with z slowly (i.e. there is little variation on the scale of a wavelength in the z direction). For these slowly varying waveguides, the beam propagation method (BPM) is widely used. These are marching methods based on approximate one-way models. Thirdly, many z-varying wave-guiding structures such as waveguide gratings, are made of piecewise z-invariant segments. The bidirectional beam propagation methods (BiBPM) are designed to take advantages of this feature. The mode matching method (MMM) is also widely used for piecewise z-invariant structures. Both BiBPM and MMM are aimed at solving the full boundary value problem while reducing unnecessary computation in each z-invariant segment. Each of these two